LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST SEMESTER 2025/2026 ACADEMIC YEAR

FOURTH YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (GENERAL)

MATH 426: ORDINARY DIFFERENTIAL EQUATIONS II

STREAM: R ***TIME: 2 HRS***

DAY: WEDNESDAY [11.30 – 1.30 P.M] ***DATE: 04/02/2026***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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Instructions: Answer question 1 (compulsory) and any other 2 (two) questions. There is no need to attempt any more questions. The following series may prove useful.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots,\ \sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots,\ \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \ldots + x^n + \ldots$$

QUESTION ONE (30 MARKS, COMPULSORY)

a) (i) Distinguish clearly between ordinary differential equations (ODE) and partial differential equations (PDE), giving a clear example of each. **(5 Marks)**

(ii) For each of the equations below, state the order and the degree

$$\frac{\partial^7 z}{\partial x^7} - \left(\frac{\partial^2 z}{\partial y^2}\right)^4 - 2xy = 9$$ **(3 Marks)**

$$\left(\frac{d^3 z}{dy^3}\right)^{\frac{1}{2}} = \frac{dz}{dx} + 1$$ **(3 Marks)**

b) (i) Define the following terms as applied in power series solutions of differential equations: analytic function, ordinary point, singular point. **(3 Marks)**

(ii) Identify all the ordinary (regular) and singular points of the ODE $(x^4 - x^2)y^{//} + (2x+1)y^{/} + x^2(x+1)y = 0$ **(3 Marks)**

c) Determine the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(-5)^n}{(n+1)}(x-10)^n$

(5 Marks)

d) Using either elimination or operator method a general solution of the system of equations

$$\frac{d^2x}{dt^2} + \frac{dy}{dt} - x + y + 1 = 0$$
$$\frac{dx}{dt} + \frac{dy}{dt} - x - t^2 = 0$$

(8 Marks)

QUESTION TWO (20 MARKS, OPTIONAL)

a) Using the series provided in the instructions section of this examination, employ long division (synthetic division) to obtain a power series for tan (x) **(4 Marks)**

b) The system of differential equations below has complex eigenvalues

$$\begin{bmatrix} x^{/} \\ y^{/} \end{bmatrix} = \begin{bmatrix} 3x & -13y \\ 5x & y \end{bmatrix}$$

(i) Determine the eigenvalues **(3 Marks)**

(ii) Determine the associated eigenvectors **(3 Marks)**

(iii) Determine the general solution of the equation **(6 Marks)**

(iv) When t = 0, $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 3 \\ -10 \end{bmatrix}$, obtain the complete solution **(4 Marks)**

QUESTION THREE (20 MARKS, OPTIONAL)

a) Consider the homogeneous system $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x & +2y \\ 2x & +y \end{bmatrix} + \begin{bmatrix} 6e^{3t} \\ 2e^{3t} \end{bmatrix}$. Solve it using,

(i) Method of Undetermined Coefficients **(9 Marks)**

(ii) Method of Variation of Parameters **(9 Marks)**

Comment on your answers. **(2 Marks)**

QUESTION FOUR (20 MARKS, OPTIONAL)

a) Consider the system of differential equations
$$\frac{d^2x}{dt^2} + \frac{dy}{dt} + y - x + 4 = 0$$
$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - x - t^3 + 1 = 0$$

Solve it by combination of operator and elimination methods. **(8 Marks)**

b) The process of reducing the order of second order linear differential equation to that of first order with dependent variable $\frac{dv}{dx}$ involves expressing $y = f(x)$ which is a known solution, as $y = v(x)f(x)$. Prove that statement. **(5 Marks)**

c) Given $\frac{d^2y}{dx^2}(x+1)^2 - 3(x+1)\frac{dy}{dx} + 3y = 0$

(i) Use intuition to come up with a first solution **(2 Marks)**

(ii) Solve the equation using the Reduction of Order approach **(5 Marks)**

QUESTION FIVE (20 MARKS, OPTIONAL)

(a) You are given the equation $\frac{d^2y}{dx^2}(x\char`^2 + 1) \quad + x\frac{dy}{dx} - y = 0.$

(i) Rewrite the equation as an infinite power series **(3 Marks)**

(ii) Harmonize counters and indices and obtain the recurrent relation **(5 Marks)**

(iii) Obtain C_1, C_2 and C_3 **(4 Marks)**

(iv) Obtain the general solution of this equation. **(3 Marks)**

b) Verify that $y = C_1e^{-x} + C_2e^{2x}$ is a solution to the homogeneous ODE $\frac{d^2y}{dy^2} - \frac{dy}{dx} - 2y = 0$ for all values of constants C_1 and C_2 **(3 Marks)**

c) Why is the series method of solving ODEs worthwhile, despite its apparent tediousness? **(2 Marks)**