



UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

**THIRD YEAR EXAMINATION FOR THE DEGREE OF
BACHELOR OF SCIENCE (STATISTICS)**

MATH 314: ORDINARY DIFFERENTIAL EQUATIONS 1

STREAM: R

TIME: 2 HRS

DAY: TUESDAY [8.30A.M – 10.30A.M]

DATE: 09/04/2024

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS**QUESTION ONE (30 MARKS)**

- (a) Explain the meaning of the following terms;
- (i) Solution of a differential equation **(2 Marks)**
- (j) Integrating factor of a differential equation **(2 Marks)**
- (b) State the order and degree of each of the following differential equations

(i) $\left(\frac{d^4 r}{ds^3}\right)^5 - 3\frac{d^2 r}{ds^2} + 2\left(\frac{dr}{ds}\right)^4 - 5r = 20\ln s$ **(2 Marks)**

(ii) $q''(x) = \sqrt{4q'''(x) + 6x^2 q}$ **(2 Marks)**

- (c) Obtain the differential equation associated with;

(i) $y = Kx^2 + Mx$ where K and M are constants **(3 Marks)**

(ii) $y = A \cos(3x + B) + Ce^{2x}$ Where A , B and C are constants **(3 Marks)**

- (d) Solve the following ordinary differential equations;

$$(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$$
 (3 Marks)

- (e) Solve the initial value problem $(x^2 + 1)\frac{dy}{dx} + 4xy = x$, $y(0) = 1$ **(3 Marks)**

- (f) Suppose a certain population of bacteria grows at a rate proportional to itself. If initially there are 10,000 organisms and the number triples after 9 hours, how long will it be before the population reaches 70 times the original population? (Round your answer to one decimal place.) **(5 Marks)**

- (g) Use the method of undetermined coefficients to solve $\frac{dy^2}{dx^2} - 2\frac{dy}{dx} + 5y = 4e^{5x}$ **(5 Marks)**

QUESTION TWO (20 MARKS)

- (a) Define a general linear differential equation of order n involving the dependent variable y and the independent variable x where $a_0(x) \neq 0, a_0(x), a_1(x) \dots a_n(x)$ are constants or functions of x . When is a differential equation said to be non-linear? **(5 Marks)**

- (b) By making the substitution $x = e^t$ find the general solution of the equation; (5 Marks)

$$x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = x^3$$

- (c) Use the method of variation of parameters to solve; $\frac{d^2 y}{dx^2} + y = \tan x$ (5 Marks)

- (d) Show that $\frac{dy}{dt} = \frac{ty}{t^2 + y^2}$ is an homogeneous function in t and y. Using the substitution $y=vt$,

transform the equation into an equation involving v and t only. Hence solve the equation.

(5 Marks)

QUESTION THREE (20 MARKS)

- (a) By use of an example, distinguish between a general solution and a particular solution (4 Marks)

- (b) Show that $y = Ae^{2x} + Be^{3x}$ is the general solution of the differential equation (4 Marks)

$$\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0$$

- (c) Solve $(ye^x + 2e^x + y^2)dx + (e^x + 2xy)dy = 0$ (4 Marks)

- (d) A certain quantity Q increases at a rate proportional to itself. If Q=12 when t=0, and Q=36 when t=2, Find Q when t=4 (4 Marks)

- (e) A breeder reactor converts uranium-238 into an isotope of plutonium-239 at a rate proportional to the amount of uranium-238 present at any time. After 10 years, 0.03% of the radioactivity has dissipated (that is, 0.9997 of the initial amount remains). Suppose that initially there is 170 pounds of this substance. Find the half-life uranium 238. (Round your answer to the nearest whole number.) (4 Marks)



QUESTION FOUR (20 MARKS)

- (a) Show that $y = e^{-x} + 2xe^{-x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$ (3 Marks)
- (b) Solve the following equations;
- (i) $\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 15x = 0$ (3 Marks)
- (ii) $\frac{dy^2}{dx^2} - 10\frac{dy}{dx} + 25y = 0$ (3 Marks)
- (iii) $\frac{d^2w}{dz^2} - 6\frac{dw}{dz} + 25w = 0$ (3 Marks)
- (c) Show that $(4x + 3y^2)dx + 2xydy = 0$ is not exact but has an integrating factor of the form x^n where n is a positive integer. Determine the value of n hence solve the equation. (4 Marks)
- (d) Solve the Bernoulli equation $\frac{dy}{dx} + y = xy^3$ (4 Marks)

QUESTION FIVE (20 MARKS)

- (a) Find the total differential of $F(x, y) = 4x^2y^3 - 2x^3y^2$ (4 Marks)
- (b) Obtain the differential equation associated with $f(x) = c_1e^{-3x} + c_2e^{4x} + c_3e^{-5x}$ (4 Marks)
- c) Solve the initial value problem $\frac{dy}{dx} = \frac{y^2 + 1}{yx^2 + y}$ $y(1)=0$ (4 Marks)
- d) Determine whether the equation $(x^3 + y^3)dx - 3xy^2dy = 0$ is homogeneous or not. Hence or otherwise solve it. (4 Marks)
- e) Solve the initial value problem; $\frac{dy}{dx} + y \cot x = \operatorname{cosec} x$ $y(0)=2$ (4 Marks)