LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST SEMESTER 2025/2026 ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (STATISTICS)

STAT 313: DESIGN AND ANALYSIS OF EXPERIMENTS

STREAM: R ***TIME: 2 HRS***

DAY: MONDAY [8.30 – 10.30 A.M] ***DATE: 02/02/2026***

THIS QUESTION PAPER CONSISTS OF SEVEN (7) PAGES

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INSTRUCTIONS: - Answer **QUESTION ONE** and **ANY OTHER TWO** questions

QUESTION ONE (30 MARKS)

(a) State and briefly explain the reasons for running experiments based on objectives of experimental designs. **(5 Marks)**

(b) Give the outline of the procedures in design of experiments **(4 Marks)**

(c) Complete the following table below for the analysis of variance and expected mean squares of an experimental design.

Source of variation	Sum of squares(SS)	Degrees of freedom	Mean sum of squares(Mss)	F-value
Treatments	?	2	4.68	
Error	?	?	?	
Total	10.96	11		

(2 Marks)

Test the hypothesis that; The treatments effects are equal to zero, Use $\alpha = 0.05$ **(2 Marks)**

(d) Show that the expectation of an error sum of squares (SSError) results to (N-k) σ^2,where N and k are respectively total observations and the number of treatments of a completely randomized design (CRD) **(4 Marks)**

(e) State the statistical model for each of the following experimental designs and explain what each of the components in the model stand for;

(i) Latin square design **(3 Marks)**
(ii) Graeco Latin square design **(3 Marks)**

(f) A chemist wishes to test the effect of four chemical agents on the strength of a particular type of cloth. Because there might be variability from one bolt of cloth to another, the chemist decides to make up five test specimen for each chemical agent level. With the tensile strength shown in the table below;

Chemicals	Bolt				
	1	2	3	4	5
1	73	68	74	71	67
2	73	67	75	72	70
3	75	68	78	73	68
4	73	71	75	75	69

(i) Using the appropriate design, prepare an ANOVA **(5 Marks)**

(ii) Does the data provide sufficient evidence to indicate whether the different chemical agents are effective or not at $\alpha = 5\%$ level of significance?
(2 Marks)

QUESTION TWO (20 MARKS)

(a) State components of a basic ANOVA variability and explain what each of the component partition stand for **(4 Marks)**

(b) What do you understand by 'Design of an experiment' **(2 Marks)**

(c) Show that in a fixed effect model the sum of squares of treatment is given as;

$$SS_{Treatment} = \sum_{i=1}^{k} \frac{y_i^2}{n_i} - \frac{y_{..}^2}{N}$$

(6 Marks)

(d) A study measures the sorption (either absorption or adsorption) rate of three different types of organic chemical solvents. These solvents are used to clean industrial fabricated-metal parts and are potentially hazardous waste. Independent samples of solvents from each type were tested and their sorption rates were recorded in the table below as a mole percentage.

Types of organic chemical solvents	Observations
Aromatics	1.06 0.95 0.79 0.65 0.82 1.15 0.89 1.12 1.05
Chloroalkanes	1.58 1.12 1.45 0.91 0.57 0.83 1.16 0.43
Esters	0.29 0.43 0.06 0.06 0.51 0.09 0.44 0.10 0.17 0.55 0.53 0.17 0.61 0.34 0.60

(i) Is there a significant difference in the mean sorption rate for the three solvents? **(6 Marks)**

(ii) Which solvent would you use with regard to the sorption rates? **(2 Marks)**

QUESTION THREE (20 MARKS)

(a) When is the Latin square better than;

(i) Completely randomized design?

(ii) Randomized block design? **(4 Marks)**

(b) Briefly describe a Latin square design **(4 Marks)**

(c) An industrial engineer is investigating the effect of five assembly methods (*A, B, C, D, and E*) on the assembly time for a color television component. Five operators are selected for the study. Furthermore, the engineer knows that each assembly method produces such fatigue that the time required for the last assembly may be greater than the time required for the first, regardless of the method. That is, a trend develops in the required assembly time. To account for this source of variability, the engineer uses an appropriate design using the data represented in the table below,

Order Of Assembly	Operator				
	1	2	3	4	5
1	14 A	22E	20B	18C	25D
2	19 B	21D	16A	23E	18C
3	23 D	15A	20C	18B	23E
4	21 C	46B	24E	21D	18A
5	23E	16C	23D	17A	19B

Does the data provide sufficient evidence to indicate a difference in?

i. Order Of Assembly
ii. Operators
iii. Assembly methods

Use $\alpha = 0.05$ level of significance. **(12 Marks)**

QUESTION FOUR (20 MARKS)

(a) Complete the following table below for the analysis of variance and expected mean squares of a Latin square design.

Source of variation	Degree of freedom	SS	MSS
treatment effect	4	?	?
Row	?	150	?
Column	?	?	17
Error	?	?	10.67
Total	?	676	

(4 Marks)

Test the hypothesis that;

(i) The treatments effects are equal to zero **(1 Mark)**
(ii) The Column effects are equal to zero. Use $\alpha = 0.05$ **(1 Mark)**

(b) Briefly explain randomization in the following designs: -

(i) Latin square design **(2 Marks)**

(ii) Completely randomized design **(2 Marks)**

(c) A manufacturer of paper used for making grocery bags is interested in improving the tensile strength of the product. It is known that tensile strength is a function of the hardwood concentration in the pulp and the experimenter is interested in concentrations between 5% and 20%. Experimenter decides to investigate four levels of hardwood concentration: 5%, 10%, 15%, and 20%. He decides to make up six **BLOCKS** for test specimen for each concentration level, using a pilot plant. The data from this experiment are shown in the Table below;

Hardwood Concentration (%)	Observations					
5%	7	8	15	11	9	10
10%	12	17	13	18	19	15
15%	14	18	19	17	16	18
20%	19	25	22	23	18	20

Test the hypothesis that different hardwood concentration do not affect the mean tensile strength of the paper at $\alpha = 0.01$ level of significance. **(10 Marks)**

QUESTION FIVE (20 MARKS)

(a) State and briefly describe the main objectives in design of experiments. **(3 Marks)**

(b) State and briefly explain the methods used in design and analysis of experiments to eliminate or reduce the variations when conducting experiments. **(6 Marks)**

(c) In the book Design of Experiments for the Quality Improvement published by the Japanese Standards Association (2007), a study on the amount of dye needed to get the best color for a certain type of a fabric was conducted. The three amounts of dye, $\frac{1}{3}\%$ wof ($\frac{1}{3}\%$ of the weight of a fabric), 1% wof, and 3% wof, were each administered at two different plants. The color density of a fabric was then observed four times for each level of dye at each plant.

	Amount of dye					
	$\frac{1}{3}\%$		1%		3%	
Plant 1	5.2	6.0	12.3	10.5	22.4	17.8
	5.9	5.9	12.4	10.9	22.5	18.4
Plant 2	6.5	5.5	14.5	11.8	29.0	23.2
	6.4	5.9	16.0	13.6	29.7	24.0

Perform an analysis of variance to test the hypothesis, at the 0.05 level of significance, that there is no difference in the color density of a fabric for the three levels of dye.

Consider plants to be blocks. **(11 Marks)**