LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST SEMESTER 2025/2026 ACADEMIC YEAR

FOURTH YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS/SCIENCE) AND BACHELOR OF SCIENCE (GENERAL)

MATH 415: TOPOLOGY I

STREAM: R ***TIME: 2 HRS***

DAY: FRIDAY [14.30 – 16.30 P.M] ***DATE: 06/02/2026***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

PLEASE DO NOT OPEN UNTIL THE INVIGILATOR SAYS SO.

INSTRUCTIONS

Answer questions **ONE** and any other **TWO** Questions.

<u>QUESTION ONE (30 MARKS)</u>

(a) Let (X,τ) be a topological space and let $A,B \subseteq X$. Then what will be ϕ^c *and* X^c respectively. (A) X *and* ϕ. (B) ϕ *and* X. (C) X *and* X. (D) ϕ *and* ϕ **(2 Marks)**

(b) If A and B are two subsets of a topological space (X,τ) and they are disjoint. Then what will be $A \cap B$?. (A) ϕ. (B) $A \cup B$. (C) X. (D) None of these. **(2 Marks)**

(c) In a topological space (X,τ) the members of τ are called?. (A) Open sets. (B) Closed sets. (C) Topology members. (D) None of the above. **(2 Marks)**

(d) Let (X,τ) be a topological space and $A \subset X$. Then exterior of A is?. (A) $X - A$. (B) $(X-A)^c$. (C) $(X-A)^{\circ}$. (D) X. **(2 Marks)**

(e) In a topological space (X,τ), the subclasses ϕ *and* X are? (A) Always open. (B) Always closed. (C) Connected. (D) None of these. **(2 Marks)**

(f) What is the relationship between $\overline{A}, \overline{B}$ *and* $\overline{A \cap B}$. (A) $\overline{A \cap B} = \overline{A} \cap \overline{B}$. (B) $\overline{A \cap B} = \overline{A} \cup \overline{B}$. (C) $A \cap B \neq \overline{A} \cup \overline{B}$. (D) None of the above. **(2 Marks)**

(g) Let X be a none empty set and $\Re$ be the set of real numbers, then $d : X * X \to \Re$ is called?. (A) Metric. (B) Metric space. (C) Connected set. (D) Closed. **(2 Marks)**

(h) Let (X,τ) be a topological space and let $A,B \subseteq X$, then what will be ϕ° *and* X° respectively?. (A) X *and* ϕ. (B) ϕ *and* X. (C) X *and* X. (D) ϕ *and* ϕ. **(2 Marks)**

(i) Let (X,τ) be a usual topological space, where $X = \{a,b,c\}$. Then which of the following is not a topology on X. (A) $\tau_1 = \{\phi, X\}$. (B) $\tau_2 = \{\phi, X, a\}$. (C) $\tau_3 = \{\phi, X, \{a\}, \{a,b\}\}$. (C) $\tau_4 = \{\phi, X, \{a\}, \{a,b\}, \{b,c\}\}$. **(3 Marks)**

(j) Let $X = \{a,b,c\}$ *and* $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$ be a topology on X. Then which of the following is not closed in X. (A) $\{b,c\}$. (B) c. (C) $\{a,c\}$. (D) None of the above. **(3 Marks)**

(k) Which of the following is a compact set? (A) $(0,1)$. (B) $[0,1]$. (C) $(1,1,)$. (D) $(0,0)$. **(2 Marks)**

(l) What will be the value of $\overline{A}$ in terms of boundary of A, $b(A)$? (A) $\overline{A} = A \cap b(A)$. (B) $\overline{A} = A \cup b(A)$. (C) $\overline{A} = A - b(A)$. (D) None. **(2 Marks)**

(m) A is open if and only if $A \cap b(A) = -$?. (A) ϕ. (B) ϕ^c. (C) X. (D) A. **(3 Marks)**

QUESTION TWO (20 MARKS)

Let (X,τ) be a topological space.

(a) Give the definition of a topological space **(3 Marks)**

(b) Let X *and* Y be topological space. Describe under which condition a function $f : X \to Y$ is said to be (i) Continuous. (ii) An identification map. **(4 Marks)**

(c) Let τ be a topology on a set X consisting of four sets $\tau = \{X, \phi, A, B\}$. With A *and* B are nonempty distinct proper subsets X. What condition must A *and* B satisfy?. **(7 Marks)**

(d) Let $X = \{a,b,c,d\}$. Determine whether or not each of the following classes of subsets of X is a topology $(i)\ \tau_1 = \{X, \phi, \{a\}, \{a,b\}, \{a,c\}\}$ $(ii)\ \tau_2 = \{X, \phi, \{a,b,c\}, \{a,b,d\}, \{a,b,c,d\}\}$. $(iii)\ \tau_3 = \{X, \phi, \{a\}, \{a,b\}, \{a,c,d\}, \{a,b,c,d\}\}$ **(6 Marks)**

QUESTION THREE (20 MARKS)

Let (X,d) be a metric space.

(a) Define the open ball $B_r(x)$ of radius $r > 0$ and centered at $x \in X$. **(2 Marks)**

(b) Define an open set $A \subset X$. **(2 Marks)**

(c) Show that the open ball $B_r(x)$ in X is an open set. **(8 Marks)**

(d) Prove that a finite union of pen sets is also open. **(8 Marks)**

QUESTION FOUR (20 MARKS)

Let (X,τ) be a topological space

(a) Define a limit point of a point say $p \in X$. **(2 Marks)**

(b) Consider the following topology on $X = \{a,b,c,d,e\}$

$\tau = \{X, \phi, \{a\}, \{a,b\}, \{a,c,d\}, \{a,b,c,d\}, \{a,b,e\}\}$.

(i) List the closed subsets of X. **(2 Marks)**

(ii) Determine the derived sets of $(i)\ A = \{c,d,e\}$ $(ii)\ B = \{b\}$. **(8 Marks)**

(b) Using the same topology in part (b) above. Determine the derived sets of $(i)\ A = \{c,d,e\}$ $(ii)\ B = \{b\}$ **(8 Marks)**

QUESTION FIVE (20 MARKS)

(a) Let (X,τ) be a topological space and $A, B \subseteq X$. Prove that

(i) A is closed if and only if $A = \overline{A}$ **(4 Marks)**

(ii) A is open if and only if $A = A^{\circ}$ **(4 Marks)**

(iii) $\overline{A \cup B} = \overline{A} \cup \overline{B}$. **(6 Marks)**

(b) State and prove the condition under which a sequence $\{x_n\}$ in a topological space converges to a unique limit. **(8 Marks)**

(c) Consider the following topology on $X = \{a, b, c, d, e\}$.
$\tau = \{X, \phi, \{a\}, \{a,b\}, \{a,c,d\}, \{a,b,c,d\}, \{a,b,e\}\}$. List the members of the relative topology τ_A on $A = \{a, c, e,\}$ **(6 Marks)**