LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

SECOND YEAR EXAMINATION FOR THE DEGREES OF BACHELOR OF EDUCATION (SCIENCE), BACHELOR OF EDUCATION (ARTS), BACHELOR OF COMPUTER SCIENCE, BACHELOR OF SCIENCE (GENERAL)

MATH 221: LINEAR ALGEBRA 1

STREAM: R ***TIME: 2 HRS***

DAY: FRIDAY[8.30 – 10.30 A.M] ***DATE: 12/04/2024***

THIS QUESTION PAPER CONSISTS OF THREE (3) PAGES

PLEASE DO NOT OPEN UNTIL THE INVIGILATOR SAYS SO.

INSTRUCTIONS: DO QUESTIONS ONE AND ANY OTHER TWO QUESTIONS.

QUESTION ONE (30 MARKS)

a) Given the matrix A = $\begin{bmatrix} -1 & 1 & 3 \\ 2 & 1 & -1 \\ 4 & 2 & 2 \end{bmatrix}$. Find

i) $Cof_{ij}(A)$ **(9 Marks)**

ii) Cofactor matrix of A **(1 Mark)**

iii) Adjoint of A **(1 Mark)**

iv) Inverse of A **(1 Mark)**

b) At what point is Cramer`s rule used to solve a system of linear equations. Hence solve the system

$CX = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad where \quad C = \begin{bmatrix} 1 & -1 & 2 \\ -3 & -3 & 3 \\ 4 & 0 & 5 \end{bmatrix}$ **(8 Marks)**

c) Determine if the following vectors from R^{3x1} are linearly independent or linearly dependent.

$v_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \; v_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}, \; v_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ **(9 Marks)**

QUESTION TWO (20 MARKS)

a) Find the eigenvalues and eigenvectors of A = $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix}$ **(10 Marks)**

b) Find a spanning set for the null space of

$A = \begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ **(10 Marks)**

QUESTION THREE (20 MARKS)

(a) Determine if the following polynomials are linearly independent or linearly dependent.

$p(x)=x^2-1,\ q(x)=x^2+x-2,\ \ r(x)=x^2+3x+2$. **(10 Marks)**

b) Let $M_1=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix},\ M_2=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix},\ M_3=\begin{bmatrix}1 & 3\\ 0 & 4\end{bmatrix},\ M_4=\begin{bmatrix}0 & 2\\ 2 & 0\end{bmatrix},\ M_5=\begin{bmatrix}1 & 1\\ 2 & 3\end{bmatrix}$

Test for linearly independent or linearly dependent. **(10 Marks)**

QUESTION FOUR (20 MARKS)

a) Determine whether V = {u in R^3 / ||u|| = 1} is a subspace of R^3 **(6 Marks)**

b) Find a basis for the subspace W of R^{4x1} spanned by

$\{u_1=[1\ \ 2\ \ 3\ \ 1], u_2=[3\ \ 2\ \ 1\ \ 1], u_3=[0\ \ 2\ \ 4\ \ 1], u_4=[1\ \ 1\ \ 1\ \ 1]\}$

Hence state the dimension of $RT\ (A)$ **(14 Marks)**

QUESTION FIVE (20 MARKS)

a) Find a basis for the column space of

$$\mathbf{A}=\begin{bmatrix}1 & 0 & 1 & 1\\ 3 & 2 & 5 & 1\\ 0 & 4 & 4 & -4\end{bmatrix}.$$

(10 Marks)

b) Express M as a linear combination of the matrices A, B, and C where

$M=\begin{bmatrix}4 & 7\\ 17 & 9\end{bmatrix},\ A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix},\ B=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\ \ and\ \ C=\begin{bmatrix}1 & 1\\ 4 & 5\end{bmatrix}$ **(10 Marks)**