LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

1ST SEMESTER 2022/2023 ACADEMIC YEAR

SECOND YEAR EXAMINATION FOR THE DEGREES OF BACHELOR OF EDUCATION (SCIENCE)/BACHELOR OF EDUCATION (ARTS)/ BACHELOR OF SCIENCE (STATISTICS)/ BACHRLOR OF SCIENCE (ECONOMICS & STATISTICS)/BACHELOR OF SCIENCE (GENERAL)

MATH 211: CALCULUS II

STREAM: R ***TIME: 2 HRS***

DAY: MONDAY [8.30AM – 10.30AM] ***DATE: 19/12/2022***

THIS QUESTION PAPER CONSISTS OF FIVE (5) PAGES

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INSTRUCTIONS: Answer Question ONE and Any Other TWO Questions

QUESTION ONE (30MARKS)

(a) What is integration? (2 marks)

(b) State the technique that you can use to integrate the following functions hence or otherwise solve them;

(i) $\int x^2 e^{2x^3+5} dx$ (4 marks)

(ii) $\int x^2 \cos x dx$ (4 marks)

(iii) $\int \frac{3}{x^2+4x-21} dx$ (4 marks)

(c) Use suitable trigonometric substitution to evaluate; (4 marks)

$$\int_0^2 \frac{5}{\sqrt{4-y^2}} dy$$

(d) Determine the area of the surface generated by rotating the curve $y = x^3, 1 \le x \le 2$ about the x-axis (4 marks)

(e) Evaluate the improper integral $\int_0^\infty \frac{3}{1+x^2} dx$ (4 marks)

(f) Use Simpson's rule with n=4 to approximate; (4 marks)

$\int_1^9 \frac{1}{x} dx$ correct to 4 significant figures

QUESTION TWO (20MARKS)

(a) Give three applications of integration (3 marks)

(b) Find;

(i) Antiderivative of $(q-2)(4+3q-q^2)$ with respect to q (3 marks)

(ii) $\int_0^1 x^2\sqrt{1-x^3}\,dx$ (3 marks)

(iii) $\int x^2 e^x dx$ (3 marks)

(c) Find the area under the curve $y=6-4x-2x^2$ between the lines $x=-2$ and $x=3$ and the $x-axis$ (Hint: Sketch the curve first) (4 marks)

(d) Compute the arc length of the graph of $f(x)=x^{\frac{3}{2}}$ for $0 \le x \le 4$. (4 marks)

QUESTION THREE (20MARKS)

(a) Determine $\int \frac{6x^5-3x^2-7x^2-17x+13}{2x^5-3x^4-2x+3}dx$ (4 marks)

(b) Given that $f''(x)=-18\cos(3x)$ and $f'(0)=0$ and $f'(0)=5$ Find $f(\frac{\pi}{6})$ (4 marks)

(c) A factory is polluting a lake in such a way that the rate of pollutants entering the lake at time t in months is given by $N'(t)=280t^{3/2}$
Where N is the total number of kilograms of pollutants in the lake at time t.

(i) How many Kilograms of pollutants enter the lake in 15 months (4 marks)

(ii) An Environmental Board tells the factory that it must begin cleanup procedures

After 50,000 kg of pollutants have entered the lake. After what length of time will this occur? (4 marks)

(d) Find the area under the curve $y=e^x \sin x$ between the lines $x=0$ and $x=\frac{\pi}{2}$ and the line $y=0$ (4 marks)

QUESTION FOUR (20MARKS)

(a) Given that $\frac{dy}{dx} = 6 - 2x$ and x=2 when y=0, find

(i) y when x=3 (2 marks)

(ii) x when y=-24 (2 marks)

(b) Find;

(i) $\int \frac{4t^{3/2} - 2t^{1/2} + t^{-3/2}}{4t^{-1/2}} dt$ (3 marks)

(ii) $\int \frac{x^2 + \cos 3x - e^{3x}}{x^3 + \sin 3x - e^{3x}} dx$ (3 marks)

(c) Use trigonometric identities to find $\int \sin 4x \cos 5x dx$ (3 marks)

(d) Find the length of the curve $y = \ln(\sec x)$ on the interval $[0, \frac{\pi}{4}]$ (3 marks)

(e) Determine the volume of solid obtained by rotating the region bounded by $y = x^2 - 4x + 5, x = 1, x = 4$ and $y = 0$ about the x-axis (4 marks)

QUESTION FIVE (20MARKS)

a) Find;

(i) $\int \sin^2(x)\cos^3(x) dx$ (3 marks)

(ii) $\int \frac{\sec(4\ln x)\tan(4\ln x)}{x} dx$ (3 marks)

b) Determine the length of the curve $x = e^t \cos t, y = e^t \sin t, 0 \le t \le \pi$ (3 marks)

c) Compute the volume of the solid obtained by revolving the region $y = 2\sqrt{x-1}$ and $y = x - 1$ about the line $x = -1$ (3 marks)

d) Use trapezoidal and Simpson's rule to approximate;

$\int_{1}^{9} \frac{2}{x+1} dx$ for n=8 correct to 7 significant figures. Compare the answers you obtained with the exact value. (4 marks)

e) In calm waters, the oil spilling from the ruptured hull of a grounded tanker forms an oil slick that is circular in shape. If the radius *r* of the circle is increasing at the rate of $r'(t) = \frac{30}{\sqrt{2t+4}}$ ft/min, *t* min after the rupture occurs;

(i) Find an expression for the radius r(t) at any time *t*. *Hint*: $r(0) = 0$ (2 marks)

(ii) How large is the polluted area 16 min after the rupture occurred?
(*Leave your answer in terms of* π) (2 marks)