LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST SEMESTER 2025/2026 ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE DEGREES OF BACHELOR OF SCIENCE (GENERAL)

MATH 316: LINEAR ALGEBRA II

STREAM: R ***TIME: 2 HRS***

DAY: THURSDAY [14.30 – 16.30 P.M] ***DATE: 29/01/2026***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS:

Answer question ONE and any other TWO

QUESTION ONE (30 MARKS)

a) Let $f(\lambda)=(\lambda^2-1)(\lambda-3)^2$ be the characteristic polynomial of A.
Evaluate

i) The determinant of A **(3 Marks)**

ii) The eigenvalues of A **(3 Marks)**

b) The linear map $T:\mathbb{R}^2\to\mathbb{R}^2$ is represented by the matrix A ;

$$A=\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}$$

i) Compute a non-zero vector $\boldsymbol{x}$ such that $A\boldsymbol{x}=3\boldsymbol{x}$ **(4 Marks)**

ii) Verify Caley Hamilton's Theorem using matrix A **(4 Marks)**

c) i) Given the matrices A and B such that

$|(AB^2)|=12$, and $|A^{-1}|=3$, calculate $|B|$ **(4 Marks)**

ii) Evaluate k for which C is an orthogonal matrix over $\mathbb{R}$ **(4 Marks)**

$$C=\begin{pmatrix}-k & 0\\ 0 & k\end{pmatrix}$$

d) i) Prove that similar matrices have equal determinants **(4 Marks)**

ii) Determine D, the diagonal matrix similar to $A=\begin{pmatrix}2 & 1\\ -3 & -2\end{pmatrix}$. **(4 Marks)**

QUESTION TWO (20 MARKS)

a) Calculate the minimal polynomial of A, where

$$A=\begin{pmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 3 & -3 & 2\end{pmatrix}$$ **(10 Marks)**

b) The matrix $A = \begin{pmatrix} 2 & 1 & 3 & 4 \\ -5 & 2 & 0 & 2 \\ -3 & 3 & 3 & 6 \end{pmatrix}$ represents a linear transformation $T: U \longrightarrow V$

Determine

i) The rank of T **(4 Marks)**

ii) Dimension of the kernel of T **(3 Marks)**

iii) The value of k for which vector $x=(1, k, 0, -1)$ is in the kernel of T **(3 Marks)**

QUESTION THREE (20 MARKS)

a) Given the matrix $A = \begin{pmatrix} 4 & -2 & -2 \\ 0 & h & 0 \\ 3 & 0 & -1 \end{pmatrix}$

i) Evaluate h for which the determinant of A is 4 **(4 Marks)**

ii) Determine the eigenvalues of A when h=–1 **(5 Marks)**

iii) Use your results in ii) to obtain the characteristic polynomial of A^2 in the form; $(\lambda-e_1)^m(\lambda-e_2)^n$, where m and n are the algebraic multiplicities of e_1 and e_2 respectively.

(5 Marks)

b) The characteristic and minimal polynomial of matrix A are; $f(\lambda) = (\lambda - 3)^4(\lambda - 2)^2$ and $m(\lambda) = (\lambda - 3)^2(\lambda - 2)^2$ respectively.

Write the Jordan canonical form of A

(6 Marks)

QUESTION FOUR (20 MARKS)

a) Determine Q, the matrix of the quadratic form;

$q(x_1, x_2, x_3)= x_1^2+7x_2^2-3x_3^2+4x_1x_2-2x_1x_3+8x_3x_2$

(6 Marks)

b) Let u be a fixed vector in $\mathbb{R}^3$ and T be the bilinear form defined by $\mathrm{T}(v) = u \cdot v$ for any vector v in $\mathbb{R}^3$

i) Show that T is a linear functional **(6 Marks)**

ii) Evaluate T(**0**) **(2 Marks)**

c) Let T: $\mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x_1, x_2, x_3) = (x_3, -x_2, x_1)$. Determine whether or not T is an orthogonal operator. **(6 Marks)**

<u>QUESTION FIVE (20 MARKS)</u>

Given the matrix

$$\boldsymbol{A} = \begin{pmatrix} -1 & 0 & 3 \\ 2 & 1 & -3 \\ 1 & 0 & 1 \end{pmatrix}$$

i) Compute the adjoint of $\boldsymbol{A}$

(7 Marks)

ii) Evaluate the scalar λ if $\boldsymbol{A}(\text{adj }(\boldsymbol{A})) = \lambda \mathbf{I}$

(5 Marks)

iii) Given the non-singular matrix $\boldsymbol{P}$ such that $\boldsymbol{PD}=\boldsymbol{AP}$ where,

$$\boldsymbol{P} = \begin{pmatrix} 0 & -3 & -1 \\ 1 & 3 & 1 \\ 0 & 1 & -1 \end{pmatrix},$$

Evaluate the diagonal matrix $\boldsymbol{D}$

(8 Marks)