LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

1ST SEMESTER 2022/2023 ACADEMIC YEAR

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (STATISTICS)

STAT 121: INTRODUCTION TO PROBABILITY AND STATISTICS II

STREAM: R ***TIME: 2 HRS***

DAY: FRIDAY [8.30AM – 10.30AM] ***DATE: 09/12/2022***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS: Answer question ONE and any other TWO questions

QUESTION ONE (30 MARKS)

a) Let the random variable X have the probability mass function $f_X(x) = \begin{cases} k(5-x), & x = 1,2,3,4 \\ 0 & elsewhere \end{cases}$.

   i) What must be the value k ?

(2 marks)

   ii) Find the cumulative distribution function, F_X , of X .and sketch its graph

(5 marks)

   iii) Compute $P(1 < X \leq 3)$

(2 marks)

b) A telemarketing executive has determined that 25 percent of people contacted will purchase the product. Suppose that 12 people are contacted about this product. Find the probability that among the 12 people contacted

   i) Exactly 3 will buy the product

   ii) From 4 to 6 will buy the product

(5 marks)

c) The distribution of the amount of gravel (in tons) sold by a particular construction supply company in a given week is a continuous random variable X with density function

$$f(x) = \begin{cases} \frac{3}{2}(1-x^2), & 0 < x < 1 \\ 0, & elsewhere \end{cases}$$

   i) What is the probability that the amount of gravel sold by the company in a given week is less than 0.25?

(2 Marks)

   ii) Find the mean and standard deviation of the amount of gravel sold by the company in a given week.

(5 Marks)

d) Let X be a random variable with MGF $M_X(t) = e^{8t+2t^2}$. Find the MGF of $Y = 3X - 6$ and hence find the probability that the value of the random variable Y is exceeds 27.

(5 Marks)

e) A gross of eggs contains 144 eggs. A particular gross is known to have 12 cracked eggs. An inspector randomly chooses 15 for inspection. Find the probability that, among the 15, at most three are cracked.

(5 Marks)

QUESTION TWO (20 MARKS)

a) Suppose that X is a random variable for which $E(X) = 6$ and $Var(X) = 10$. Find

   i) $E[(2 + X)^2]$

   ii) $Var(4X - 5)$

(6 Marks)

b) A garage uses a particular spare part at an average rate of 5 per week. Assuming that usage of this spare part follows a Poisson distribution, find the probability that

i) Exactly 5 are used in a particular week

(3 Marks)

ii) At least 9 but less than 12 are used in a 3-week period

(6 Marks)

c) Suppose that a researcher goes to a small college of 200 lecturers, 12 of which have blood type O-negative. She obtains a simple random sample of $n = 20$ of the lecturers. Find the mean and standard deviation of the number of randomly selected lecturers that will have blood type O-negative.

(5 Marks)

QUESTION THREE (20 MARKS)

a) The annual salaries of employees in a large company are approximately normally distributed with a mean of $50,000 and a standard deviation of $15,000. Find the probability that the annual salary of a randomly selected employee would be

i) Less than $70,000

(4 Marks)

ii) More than $40,000

(4 Marks)

iii) Between $45,000 and $67,000

(6 Marks)

b) Each customer entering a certain store will make a purchase with probability 0.6. find the probability that, among 1000 customers at least 608 but not less than 625 customers make purchases.

(6 Marks)

QUESTION FOUR (20 MARKS)

a) A rifleman fires at a target 3 times and the probability is 0.7 that he hits the target each time. Let X denote the of times he hits the target. Find the probability distribution of the random variable X .

(5 Marks)

b) A random variable X has its moment generating function given by

$$M(t)=\frac{1}{5}\left(e^{t}+2e^{4t}+2e^{8t}\right) \text{ for } -\infty < t < \infty,$$

i) Find μ and σ^2

ii) Identify the probability mass function of X

.

(9 Marks)

c) The life in years of a certain type of electrical switch has an exponential distribution with an average life of 2 years. If 10 of these switches are installed in different systems, what is the probability that at most 3 fail during the first year?

(7 marks)

QUESTION FIVE (20 MARKS)

a) Let X be a continuous variable with density function $f(x)=\begin{cases} ax+bx^2, & 0\le x\le 1 \\ 0 & elsewhere \end{cases}$. Find the values of a and b if the mean of X is $\frac{3}{5}$.

(6 Marks)

b) Let the random variable X have the probability density function $f(x)=\begin{cases} x, & 0<x<1 \\ 2-x, & 1\le x<2 \\ 0 & elsewhere \end{cases}$.

Compute the mean and variance of X.

(9 Marks)

c) If X is a random variable such that $E(X)=75$ and $E(X^2)=5650$, use Chebyshev's inequality to determine a lower bound for the probability $P(65<X<85)$.

(5 Marks)