LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (GENERAL) BACHELOR OF EDUCATION (ARTS)/BACHELOR OF SCIENCE (ECONOMICS & STATISTICS)

MATH 121: CALCULUS 1

STREAM: R ***TIME: 2 HRS***

DAY: WEDNESDAY [2.30P.M – 4.30P.M] ***DATE: 17/04/2024***

THIS QUESTION PAPER CONSISTS OF FIVE (5) PAGES

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INSTRUCTIONS: Answer Question ONE and Any Other TWO Questions

QUESTION ONE (30 MARKS)

a) Define the following terms

(i) Function

(ii) Image

(iii) Range **(3 Marks)**

b) Evaluate the following limits;

(i) $\lim_{x\to 2} 3x^2 - 4x + 7$ **(2 Marks)**

(ii) $\lim_{h\to 5} \dfrac{h^2 - 3h - 10}{h - 5}$. **(3 Marks)**

(iii) $\lim_{y\to\infty} \dfrac{4y^3 - 3y^2 + 5y - 9}{5y^3 + 2y^2 - 6y + 3}$ **(2 Marks)**

c) Given $f(x) = \dfrac{2x+1}{x-1}$ and $g(x) = \dfrac{x+1}{x-2}$ show that $fog(x) = gof(x) = x$ **(3 Marks)**

d) Find the inverse of the function $f(x) = (x-2)^5 + 3$ **(3 Marks)**

e) By taking values of x sufficiently close to $a = 2$,show that

$$\lim_{x\to 2} \frac{x^3 - 8}{x - 2} = 12$$ **(3 Marks)**

f) Differentiate the following functions by the means of the first principles

(i) $y = \sin x$ **(3 Marks)**

(ii) $y = \sqrt{x}$ **(3 Marks)**

g) An object moving in a straight line has its displacement s meters from an origin 0 at time t seconds given by $s = t(t-3)^2$.Determine

(i) Time when the object is at the origin **(2 Marks)**

(ii) Time when the object is at rest **(2 Marks)**

(iii) Distance between $t = 0$ and $t = 2$ **(2 Marks)**

QUESTION TWO (20 MARKS)

a) Giving the three conditions of continuity, define continuity of a function. Hence discuss the continuity of the following function $f(x) = \begin{cases} \sqrt{x+2}\,, & x < 2 \\ x^2 - 2\,, & 2 \leq x < 3 \\ 2x\,, & x \geq 3 \end{cases}$

at the point $x = 2$ **(10 Marks)**

b) Evaluate,

(i) $\lim_{h \to 0} \dfrac{h}{\sqrt{3h+4} - 2}$ **(3 Marks)**

(ii) $\lim_{x \to 7} (9+x)^{\frac{3}{2}}$ **(3 Marks)**

c) A student is drinking a soda from a cylindrical can through a straw.The volume of the soda in the can is decreasing at a rate of $0.5cm^3/s$.If the radius across the top of the can is $1cm$.How fast is the level of the soda in the can going down. **(4 Marks)**

QUESTION THREE (20 MARKS)

a) Define the following types of discontinuity

i) Removable discontinuity

ii) Jump discontinuity

iii) Infinite discontinuity **(3 Marks)**

b) Given the function $f(x) = \begin{cases} 2x+5, & x<-1 \\ x^2+2, & x>-1 \\ 5, & x=-1 \end{cases}$ .Determine whether it is continuous or discontinuous and if discontinuous determine which type of discontinuity. **(5 Marks)**

c) Differentiate the following functions

i) $f(x) = 7x^3 - 6x^2 + 4x + 2$ **(2 Marks)**

ii) $f(x) = (1-6x^3)(4x^2 - 6x + 2)$ **(2 Marks)**

iii) $f(x) = (x^2+3x)^7$ **(2 Marks)**

iv) $f(x) = \dfrac{x}{x-9}$ **(2 Marks)**

d) Approximate $\sqrt{17}$ using differentials **(4 Marks)**

QUESTION FOUR (20 MARKS)

a) Show that the derivative of the function a^x is $a^x \ln a$. Hence find the derivative of $y = x^x$. **(5 Marks)**

b) Find the equation of the normal to the curve $y = (x^2 + x + 1)(x-3)$ at the point where it cuts the $x-$axis **(5 Marks)**

c) Find the gradient of the curve $x = \dfrac{t}{1+t}$ and $y = \dfrac{t^3}{1+t}$ at the point $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$. **(4 Marks)**

d) Find the Third derivative for $y = 5 + 24x - 9x^2 - 2x^3$ **(3 Marks)**

e) A spherical balloon is blown up so that its volume increases at a constant rate of $2cm^3/\sec$. Find the rate of increase of radius when volume of balloon is $50cm^3$ **(3 Marks)**

QUESTION FIVE (20 MARKS)

a) Evaluate:

i) $\lim_{h\to 0}\dfrac{1-\cos 3h}{\cos 2h-1}$ **(3 Marks)**

ii) $\lim_{y\to 0}\dfrac{\tan(x+y)-\tan x}{y}$ **(3 Marks)**

b) The quantity of milk processed in a plant during an 8-hour working day is given by , $Q=120t+24t^2-2t^3$,where t represents hours worked. When is the rate of production at maximum? Use the second derivative test. **(5 Marks)**

c) Sketch the curve $y=4x^3-3x^4$ **(5 Marks)**

d) Find the slope of the curve $x^2+2xy-2y^2+x=2$ at point (-4,1) **(4 Marks)**