

UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE DEGREE OF
BACHELOR OF SCIENCE (STATISTICS)

STAT 322: HYPOTHESIS TESTING

STREAM: R

TIME: 2 HRS

DAY: MONDAY [8.30 – 10.30 A.M]

DATE: 15/04/2024

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS**QUESTION ONE (30 MARKS)**

- a) Define the following terms; **(4 Marks)**
- i) a Type I error
 - ii) a Type II error
 - iii) power of test
 - iv) Size of test
- b) The heights of maize and bean seedlings at a local farm are known to be normally distributed with $\delta_{maize} = 0.5cm$ and $\delta_{beans} = 0.8cm$. A random sample of 25 maize seedlings had a mean height of 4.8cm and a random sample of 30 bean seedlings had a mean height of 4.5cm. Is there any evidence that the mean height of maize seedlings is greater than that of bean seedlings at $\alpha = 5\%$? **(5 Marks)**
- c) The average length of a random sample of 20 rolls of wire is 2031m with a standard deviation of 47m. Test at $\alpha = 1\%$ whether the mean is greater or less than 2000m **(5 Marks)**
- d) It is claimed that sports-car owners drive on the average 18,000 miles per year. A consumer firm believes that the average mileage is probably lower. To check, the consumer firm obtained information from 40 randomly selected sports-car owners that resulted in a sample mean of 17,463 miles with a sample standard deviation of 1348 miles. What can we conclude about this claim? Use $\alpha = 0$ **(5 Marks)**
- e) In a certain factory there are two independent processes manufacturing the same item. The average weight in a sample of 250 items produced from one process is found to be 120 Ozs, with a standard deviation of 12 Ozs, while the corresponding figures in a sample of 400 items from the other process are 124 Ozs with a standard deviation of 14 Ozs. Is the difference between the two sample means significant? **(5 Marks)**
- f) A random variable X is believed to follow an $Exp(\lambda)$ distribution. In order to test the null hypothesis $\mu = 20$ against the alternative hypothesis $\mu = 30$, where $\mu = 1/\lambda$, a single value is observed from the distribution. If this value of X is less than k , H_0 is accepted, otherwise H_0 is rejected.
- i) Calculate the value of k that gives a test of size 5%. **(3 Marks)**
 - ii) Calculate the probability of a Type II error **(3 Marks)**



QUESTION TWO (20 MARKS)

State and prove Neyman-Pearson lemma

(20 Marks)**QUESTION THREE (20 MARKS)**

a) A random sample of 10 boys had the following I.Q's: 70, 120, 110, 101, 88, 83, 95, 98, 107, and 100. Do these data support the assumption of a population mean I.Q of 100? Find the reasonable range in which most of the mean I.Q values of samples of 10 boys lie **(12 Marks)**

b) Let X_1, \dots, X_n denote an independent random sample from a population with a Poisson distribution with mean λ . Derive the most powerful test for testing $H_0 : \lambda = 2$ versus $H_a : \lambda = 1/2$.

(8 Marks)**QUESTION FOUR (20 MARKS)**

a) A psychologist claims that the mean age at which children start walking is 12.5 months. Carol wanted to check if this claim is true. She took a random sample of 18 children and found that the mean age at which these children started walking was 12.9 months with a standard deviation of .80 month. It is known that the ages at which all children start walking are approximately normally distributed. Find the p -value for the test that the mean age at which all children start walking is different from 12.5 months. What will your conclusion be if the significance level is 1%? **(8 Marks)**

b) A random variable X is believed to follow an $Exp(\lambda)$ distribution. In order to test the null hypothesis $\mu = 20$ against the alternative hypothesis $\mu = 30$, where $\mu = \frac{1}{\lambda}$, a single value is observed from the distribution. If this value is less than 28, H_0 is accepted, otherwise H_0 is rejected. Calculate the probabilities of:

i) a Type I error **(3 Marks)**

ii) a Type II error **(3 Marks)**

iii) Comment on how the power is affected by the value of μ . **(3 Marks)**

c) By giving examples, differentiate between Simple and composite hypothesis **(3 Marks)**



QUESTION FIVE (20 MARKS)

- a) What is a critical region? **(2 Marks)**
- b) A random sample of size n from a normal population with $\sigma^2 = 1$ is to be used to test the null hypothesis $\mu = \mu_0$ against the alternative hypothesis $\mu = \mu_1$ where $\mu_1 > \mu_0$. Use the Neyman-Pearson lemma to find the most powerful critical region of size α . **(10 Marks)**
- c) In a random sample of 100 men are taken from a village A, 60 were found to be consuming alcohol. In other sample of 200 men are taken from village B, 100 were found to be consuming alcohol. Do the two villages differ significantly in respect of the proportion of men who consume alcohol? **(8 Marks)**

