LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (STATISTICS)

MATH 316: LINEAR ALGEBRA II

STREAM: R ***TIME: 2 HRS***

DAY:THURSDAY [11.30A.M – 1.30P.M] ***DATE: 11/04/2024***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS:

Answer question ONE and any other TWO

Stream: BSc. Statistics

QUESTION ONE (30 MARKS)

a) Given that *det* (A^2B)=8 and *det* B^{-1}=2*k*

Evaluate

i) $|A|$ when k=1 **(3 Marks)**

ii) k for which $|A|=4\sqrt{2}$ **(3 Marks)**

b i) Evaluate the minimal polynomial of $C = \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$ **(4 Marks)**

ii) Compute the vector x in $\Re^2$ if $C\mathbf{x}=(3, -6)$ **(4 Marks)**

c) The linear map $T: \mathbb{R}^2 \longrightarrow \mathbb{R}$ is defined by $T(x_1, x_2) = (x_2 - x_1)$

i) Compute $T(3\mathbf{x})$ if $T(2\mathbf{x}+\mathbf{y})=6$ and $T(\mathbf{y})=4$ **(4 Marks)**

ii) Evaluate $\mathbf{x}$, when $T(\mathbf{x})=3$ and $\|\mathbf{x}\|=\sqrt{5}$ **(4 Marks)**

d) Given the matrix $A=\begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix}$

i) Evaluate the eigenvalues and eigenvectors of A **(4 Marks)**

ii) Obtain the diagonal matrix, D in i), which is similar to A **(4 Marks)**

QUESTION TWO (20 MARKS)

Given the matrix $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$

i) Investigate whether A is diagonalizable or not **(12 Marks)**

ii) Evaluate (A+2I)(4I-A) **(8 Marks)**

QUESTION THREE (20 MARKS)

a) i) Calculate all the eigenvalues of A, where

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

(3 Marks)

ii) Evaluate the minimal polynomial of the given matrix in i) **(7 Marks)**

b) The linear transformation $T: U \longrightarrow V$ is represented by the matrix A below

$$A = \begin{pmatrix} 2 & 1 & 3 & 4 \\ -5 & 2 & 0 & 2 \\ -3 & 3 & 3 & 6 \\ -1 & 4 & 6 & 10 \end{pmatrix}$$

Determine

i) The rank of A **(5 Marks)**

ii) Dimension of the kernel of T **(3 Marks)**

iii) The value of k for which vector x=(0, 2, k, -2) is in the kernel of T **(2 Marks)**

QUESTION FOUR (20 MARKS)

a) Let T: $\mathbb{R}^3 \rightarrow \mathbb{R}^3$ be defined by $T(x_1, x_2, x_3) = (-x_3,\ x_2,\ x_1)$.

i) Evaluate T(**0**) **(2 Marks)**

ii) Determine whether T is an orthogonal operator or not

(6 Marks)

b) The characteristic and minimal polynomials of matrix A are
$f(\lambda) = (\lambda - 3)^4(\lambda + 2)^2$ and $m(\lambda) = (\lambda - 3)^2(\lambda + 2)^2$ respectively.
Write the Jordan canonical form of
A

(6 Marks)

c) Let u be a fixed vector in $\mathbb{R}^3$ and T be a transformation defined by $\mathrm{T}(v) = u \cdot v$ for any vector v in $\mathbb{R}^3$

Show that T is a linear functional **(6 Marks)**

QUESTION FIVE (20 MARKS)

Given the matrix

$$A = \begin{pmatrix} -1 & 1 & 2 \\ 2 & -3 & -4 \\ 1 & -2 & 3 \end{pmatrix}$$

i) Compute the determinant of A **(5 Marks)**

ii) Obtain the adjoint of A **(10 Marks)**

iii) Hence, evaluate the inverse of A **(5 Marks)**