

UNIVERSITY EXAMINATIONS

FIRST SEMESTER 2025/2026 ACADEMIC YEAR

**FIRST YEAR EXAMINATION FOR THE DEGREES OF
BACHELOR OF SCIENCE (GENERAL), BACHELOR OF
SCIENCE (ECONOMICS AND STATISTICS) AND
BACHELOR OF EDUCATION**

MATH 113/114: GEOMETRY AND LINEAR ALGEBRA

STREAM: R

TIME: 2 HRS

DAY: FRIDAY [8.30 – 10.30 A.M]

DATE: 06/02/2026

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

PLEASE DO NOT OPEN UNTIL THE INVIGILATOR SAYS SO.

INSTRUCTIONS TO CANDIDATES:

Answer **ALL** questions in Section **A** and any **TWO** questions in Section **B**.

SECTION A: 30 MARKS**QUESTION ONE (30 MARKS)**

a) If $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 4 & 5 \\ 1 & 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 0 \\ 5 & 9 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} 8 & 0 & 2 \\ 1 & 3 & 5 \\ 2 & 1 & 6 \end{bmatrix}$, find

(i) $A - B + C$ (2 Marks)

(ii) $A - (B + C)$ (2 Marks)

b) Simplify the complex number, $\frac{7-8i}{4+5i}$ (3 Marks)

c) Solve the expression $x^2 - 3x + 4 = 0$ (2 Marks)

d) Find the centre and radius of the circle given by $x^2 + y^2 - 2x + 4y = 8$ (3 Marks)

e) For a hyperbola $9x^2 - 16y^2 = 144$, find its vertices, foci and eccentricity. (4 Marks)

f) Solve the following system of equations using Cramer's rule. (5 Marks)

$$x + 2y + 5z = 23$$

$$3x + y + 4z = 20$$

$$6x + y + 7z = 47$$

g) Let $\vec{a} = 8\hat{i} + 2\hat{j} + 4\hat{k}$ and $\vec{b} = -3\hat{i} + 4\hat{j} - 5\hat{k}$. Find;

(i) Direction cosine of $\vec{b}$ (2 Marks)

(ii) Unit vector in the direction of $\vec{a}$ (2 Marks)

(iii) The angle between $\vec{a}$ and $\vec{b}$ (3 Marks)

h) Obtain the parametric equations of the line joining the points $(1, -4, 4)$ and $(-3, 2, -3)$.

(2 Marks)

SECTION B**QUESTION TWO (20 MARKS)**

a) Express each of the following complex numbers in polar form:

(i) $6i$ (2 Marks)

(ii) $\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)^7$ (3 Marks)

b) Find the fourth root of $1 + i\sqrt{3}$ (4 Marks)

c) Given that $z_1 = 1 + i$, and $z_2 = 1 - i\sqrt{3}$. Find the modulus and argument of the complex number $z_1 = 1 + i$. Hence, find in the form $z = [r, \theta]$ the complex numbers generated from;

(i) $z_1 z_2$ (2 Marks)

(ii) $\frac{\bar{z}_1}{z_2}$ (3 Marks)

(iii) z_2^3 (4 Marks)

QUESTION THREE (20 MARKS)

a) Find the transpose and inverse of the matrix $A = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$ (3 Marks)

b) Find the inverse of the matrix given by $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ using co-factor method. (12 Marks)

c) Using Cramm's rule, find the solution to the following system of linear equations

$$\begin{aligned} x_1 + 2x_3 &= 6 \\ -3x_1 + 4x_2 + 6x_3 &= 30 \\ -x_1 - 2x_2 + 3x_3 &= 8 \end{aligned} \quad (5 \text{ Marks})$$

QUESTION FOUR (20 MARKS)

- a) Find the centre and radius of the circle given by; $x^2 + y^2 - 4x - 6y + 4 = 0$, then graph the circle. **(4 Marks)**
- b) The equation of a conic section is given by $4x^2 - y^2 - 8x + 8$. By writing the equation in standard form, identify the conic section and state its centre. **(4 Marks)**
- c) Find the centre, eccentricity, foci, directrices and the lengths as well as the equation of the axes of the ellipse: $5x^2 + 9y^2 + 10x - 36y - 4 = 0$ **(12 Marks)**

QUESTION FIVE (20 MARKS)

- a) Find the parametric equations for the line that passes through the point $(5,1,3)$ and parallel to the vector $\hat{i} + 4\hat{j} - 2\hat{k}$. **(2 Marks)**
- b) Calculate the distance between the point $M = (1,1,3)$ and the line **(4 Marks)**

$$\frac{x-3}{4} = \frac{y+1}{2} = z-3$$

- c) Find an equation of the plane that passes through the point $(2,4,-1)$ with normal vector $n = \langle 2,3,4 \rangle$. Find the intercept and sketch the plane. **(4 Marks)**
- d) Find an equation of the plane that passes through the points $P(1,3,2)$, $Q(3,-1,6)$ and $R(5,2,0)$. **(5 Marks)**
- e) Find the angle between the planes $x + y + z = 1$ and $x - 2y + 3z = 1$ **(3 Marks)**
- f) Calculate the distance between the parallel planes; $10x + 2y - 2z = 5$ and $5x + y - z = 1$. **(2 Marks)**