LAIKIPIA UNIVERSITY

# UNIVERSITY EXAMINATIONS

## 1ST SEMESTER 2022/2023 ACADEMIC YEAR

## FIRST YEAR EXAMINATION FOR THE DEGREES OF BACHELOR OF EDUCATION(SCIENCE)

## MATH 113: GEOMETRY AND LINEAR ALGEBRA

***STREAM: R*** ***TIME: 2 HRS***

***DAY: WED [8.30AM – 10.30AM]*** ***DATE: 21/12/2022***

**THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES**

**PLEASE DO NOT OPEN UNTIL THE INVIGILATOR SAYS SO.**

## QUESTION ONE (30 MARKS)

(a) The following equations represent some conic sections. By completing the square identify each of the conic section and state its three main characteristics.

i. $16y^2 - 72x - 9x^2 - 96y = 144$ (4 marks)

ii. $16 = 16x + 36y - 9y^2 - 4x^2$ (4 marks)

(b) If $\vec{a} = 2\hat{i} + 3\hat{j} - 5\hat{k}$ and $\vec{b} = 6\hat{i} - t\hat{j} + 3\hat{k}$

i. Find $t$ if $\vec{a} \times \vec{b} = -\hat{i} - 36\hat{j} - 22\hat{k}$ (3 marks)

ii. Find the angle between $\vec{a}$ and $\vec{b}$. (3 marks)

iii. If $\vec{a}$ and $\vec{b}$ forms adjacent side of a parallelogram, find the area of the parallelogram. (2 marks)

(c) Derive the equation of the plane through the point $(x_1, y_1, z_1)$ and having perpendicular vector $N = \vec{A}\hat{i} + \vec{B}\hat{j} + \vec{C}\hat{k}$. (3 marks)

(d) Find the point in which the line $x = 1 + 2t, y = 1 + 5t, z = 3t$ meets the plane $x + y + z = 2$.

(3 marks)

(e) Find $\left[2i\left(1 + i\sqrt{3}\right)\right]^6$ using De Moivre's theorem. (3 marks)

(f) Given the matrix;

$$A = \begin{bmatrix} 3 & 1 & -4 \\ 2 & 5 & 6 \\ 1 & 4 & 8 \end{bmatrix}$$

Use the cofactor method to find $A^{-1}$. (5 marks)

## QUESTION TWO (20 MARKS)

(a) Obtain the Cartesian and Parametric equations of the line joining the points $(1,-4,4)$ and $(-3,2,-3)$. (4 marks)

(b) Find the distance from the point $(2,1,-1)$ to the line $x = 2t, y = 1 + 2t, z = 2t$. (5 marks)

(c) Calculate the distance from the point $(2,2,3)$ to the plane $2x + y + 2z = 4$. (5 marks)

(d) Find the parametric equations of the line of intersection of the planes $3x-6y-2z=3$ and $2x+y-2z=2$. (6 marks)

**QUESTION THREE (20 MARKS)**

(a) Express the following in standard form;

(i) $i^{11}$ (2 marks)

(ii) $\frac{2-i}{2+3i}$ (3 marks)

(iii) $\frac{1}{(1+i)^4}$ (2 marks)

(b) Find the fourth root of $1+i\sqrt{3}$ (4 marks)

(c) Use elementary row operations to compute the inverse of the matrix

$$A=\begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$ (5 marks)

(d) Prove that if $z_1 z_2 = r_1 r_2[\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)]$ (4 marks)

If $2_1 = r_1 \cos Q_1 + iSinQ_1$ and

$Z_2 = r_2 \cos Q_2 + iSinQ_2$

**QUESTION FOUR (20 MARKS)**

(a) Determine the centre and radius of the circle given by the equation

$x^2+y^2+2x-16y+61=0$ (4 marks)

(b) The equation of an ellipse is given by $9x^2+16y^2=576$. Find the; (notes)

i. Length of semi-major axis. (2 marks)

ii. Length of semi-minor axis. (1 mark)

(c) Find the equation of the parabola whose coordinates of the vertex and focus are $(9,6)$ and $(9,5)$ respectively. (3 marks)

(d) The equation of a hyperbola is given by $9x^2 - 4y^2 - 36x - 40y - 388 = 0$. Write the equation in standard form, then find the following;

(i) Centre. (2 marks)

(ii) Foci. (2 marks)

(iii) Vertices. (2 marks)

(iv) Equation of the asymptotes. (2 marks)

(v) Sketch the curve. (2 marks)

## QUESTION FIVE (20 MARKS)

(a) Express the given fraction in the form of $x + iy$ where $x, y \in \Re$. (cat1)

$$\frac{i}{1+\sqrt{-3}}$$ (3 marks)

(b) Determine the square root of $z = 4\left[\cos 90^\circ + i\sin 90^\circ\right]$ (3 marks)

(c) Find the inverse of the matrix by cofactor method $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix}$

Hence solve the system of linear equations. (8 marks)

$$x + 2y + 3z = 4$$
$$x + 3y + 5z = 2$$
$$x + 5y + 12z = 7$$

(d) By using Crammer's rule, solve the following system of equations (exam)

$$x + y - z = 6$$

$$3x - 2y + z = -5$$

$$x + 3y - 2z = 14$$ (6 marks)