LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

1ST SEMESTER 2022/2023 ACADEMIC YEAR

FOURTH YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

MATH 420/411: PARTIAL DIFFERENTIAL EQUATIONS I

STREAM: R ***TIME: 2 HRS***

DAY: TUESDAY [8.30AM – 10.30AM] ***DATE: 06/12/2022***

THIS QUESTION PAPER CONSISTS OF THREE (3) PAGES

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QUESTION ONE – COMPULSORY (30 marks)

(a) By eliminating the constants, generated partial differential equations from the equation below $z = (x-\propto)^2 + (y-\beta)^2$

(5 marks)

(b) Solve the following 'separable' PDE: $3\sqrt{q} + 2\sqrt{p} = 6x + 2y$

(6 marks)

(c) Find the integral surface of the Lagrange equation $x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z$ which also satisfies $x+y=0$, $z=1$

(6 marks)

(d) Explain clearly how the intersection of two surfaces $F(x,y,z)=0$ and $G(x,y,z)=0$ intersect, they generate the equations $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$, where P,Q and R are the respective Jacobians $\frac{\partial(F,G)}{\partial(y,z)}$, $\frac{\partial(F,G)}{\partial(z,x)}$ and $\frac{\partial(F,G)}{\partial(x,y)}$

(5 marks)

(e) Use the results of (b) above to recover the intersecting curves for the ODEs below

(i). $\frac{dx}{y(x+y)-az}=\frac{dy}{x(x+y)-az}=\frac{dz}{z(x+y)}$ (4 marks)

(ii). $\frac{dx}{x(y-z)}=\frac{dy}{y(z-x)}=\frac{dz}{z(x-y)}$ (4 marks)

QUESTION TWO – OPTIONAL (20 marks)

a) What is the difference between a linear and a non-linear PDE? Give an example of each. (3 marks)

b) Solve the following 2-variable Pfaffian ODE using the given hint.
$(x+2y)dx + xdy = 0$ (By determining the integrating factor) (6 marks)

c) (i)When solving the standard non-linear PDE $F(x,y,z,p,q)=0$, using the Charpit's equations, prove that if x and y are missing in the equation, the consequence is that $p = aq$ (4 marks)

(ii) Use this information to solve the PDE $q(3+p) = 2pz$ (7 marks)

QUESTION THREE- OPTIONAL (20 marks)

a) Consider the Lagragean partial differential equation $2y(z-3)p+(2x-z)q=y(2x-3)$

i) Write the subsidiary equations of this PDE (2 marks)

ii) Solve the subsidiary equations to obtain, $f(x,y,z)=c_1$ and $g(x,y,z)=c_2$ (6 marks)

iii) Obtain the general solution of this PDE (3marks)

b) Using either the Charpit or Jacobi methods, find the complete integral of the non-linear PDE $p^2 x + q^2 y = z$

(9 marks)

QUESTION FOUR- OPTIONAL (20 marks)

a) Define a homogeneous function $P(x,y, z)$ and hence a homogeneous Pfafffian Differential equation $Pdx+Qdy+R\ dz=0$

(5marks)

b) Consider the Pfaffian DE $yz(y+z)dx + xz(x+z)dy + xy(x+y)dz = 0$

i) Is this equation homogeneous? (3marks)

ii) By rewriting the equation in vector form, identify $\vec{X}$ and find $curl\ \vec{X}$

(3marks)

iii) Compute $\vec{X}\ .\ curl\ \vec{X}$ and comment on the integrability of this equation.

(4marks)

iv) Solve this equation (5marks)

QUESTION FIVE- OPTIONAL (20 marks)

a) Prove that when one variable, z, is separable in a Pfaffian DE $P\,dx + Qdy + R\,dz = 0$ (let it be z), then the part not containing the separate variable, in this case $P\,dx + Qdy = 0$ is automatically exact. (7 marks)

b) Prove clearly that to obtain a system of surfaces orthogonal to $f(x, y, z) = c$, we need to solve $\dfrac{dx}{f_x} = \dfrac{dy}{f_y} = \dfrac{dz}{f_z}$, and use this result to find a system of surfaces orthogonal to $\dfrac{z}{(3z+1)}(x+y) = c$ (13 marks)