

UNIVERSITY EXAMINATIONS

1ST SEMESTER 2022/2023 ACADEMIC YEAR

**SECOND YEAR EXAMINATION FOR THE DEGREE OF
BACHELOR OF SCIENCE (GENERAL)**

MATH 221: LINEAR ALGEBRA

STREAM: R

TIME: 2 HRS

DAY: FRIDAY [8.30AM – 10.30AM] DATE: 09/12/2022

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS: - Answer question ONE and ANY other TWO.

QUESTION ONE (30 MARKS)

- (a) The vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w} \in \mathbb{R}^3$ are as provided; $\mathbf{u}=(1, k, -1)$, $\mathbf{v}=(-1, 1, -1)$ and $\mathbf{w}=(k, -k, 1)$
- Evaluate k such that $\mathbf{w} \cdot (\mathbf{u}-\mathbf{v})=0$ (4 marks)
 - Calculate $\|2\mathbf{u}-\mathbf{v}\|$ when the value of k is 1 (4 marks)
 - Express $\mathbf{u}$ as a linear combination of $\mathbf{v}$ and $\mathbf{w}$ when $k=-1$ (3 mks)
- (b) Given the sets, $V=\mathbb{R}^3$ and $S=\{(x_1, x_2, x_3): x_3=x_2-x_1\}$
- Determine the value of h for which the vector $\mathbf{w}=(h, 2, -1)$ is in S (2 marks)
 - Investigate whether the set S is a subspace of V or not (4 marks)
- (c) Given the system of equations below;
- $$\begin{aligned}\lambda x_1 + 2x_2 &= 2 \\ 2x_1 + \lambda x_2 &= 2\end{aligned}$$
- Solve the system when $\lambda=3$ (3 marks)
 - Obtain the value of λ for which the system has no solution (4 marks)
- (d) Show that T is a linear map, if
 $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ is defined by $T(x_1, x_2, x_3) = (x_3 - x_1, x_2 + 2x_1)$
 (6 marks)

QUESTION TWO (20 MARKS)

- (a) Compute the value of h for which the following system of linear equations
- $$\begin{aligned}x_1 + x_2 - 2x_3 &= h \\ 2x_1 - hx_2 + 3x_3 &= 2 \\ 3x_1 + hx_3 &= 3,\end{aligned}$$
- has
- More than one solution
 - No solution
 - Unique solution (10 marks)
- a) The set $S=\{\mathbf{u}=\{1, 2, 0\}, \mathbf{v}=\{2, 1, c\}, \mathbf{w}=\{-1, 1, 2\}\}$ spans U , a subspace of $\mathbb{R}^3$
- Calculate the value of c for which dimension U is 2 (4 marks)
 - Solve for the scalars m and n such that $\mathbf{x}=(3, 3, -2)=m\mathbf{u} + n\mathbf{w}$ (3 marks)

- iii) Determine whether x is in U or not (2 marks)

QUESTION THREE (20 MARKS)

- (a) The map $T: \mathfrak{R}^3 \rightarrow \mathfrak{R}^2$ is defined by $T(x_1, x_2, x_3) = (2x_1, -x_3)$
- i) Determine whether T is a linear transformation (5 marks)
- ii) Compute $\mathbf{u}$ in $\mathfrak{R}^3$ such that $T(\mathbf{u}) = (2, -3)$ and $\|\mathbf{u}\| = 14$ (5 marks)
- (b) A linear transformation $T: U \rightarrow V$, from vector space U to V is represented by the matrix below

$$T = \begin{pmatrix} -1 & 2 & 1 & 0 \\ 1 & 1 & 2 & 1 \\ 3 & 0 & 3 & 2 \\ 2 & -1 & 1 & 1 \end{pmatrix}$$

- i) Compute the rank of T (5 marks)
- ii) Evaluate the dimension of the kernel of T (2 marks)
- iii) Obtain k , if $\mathbf{u} = (3, 1, 1, k)$ is in the kernel of T (2 marks)

QUESTION FOUR (20 MARKS)

- a) Determine whether the set S is a subspace of V over $\mathfrak{R}$
- i) $S = \{(x_1, x_2): x_1 = 0, x_2 \in \mathfrak{R}\}, V = \mathfrak{R}^2$
- ii) $S = \{(x_1, x_2): x_1 = 1, x_2 \in \mathfrak{R}\}, V = \mathfrak{R}^2$ (10 marks)
- b) The set $\{\mathbf{u} = (1, 2, 1), \mathbf{v} = (1, c+1, 1), \mathbf{w} = (-2, -4, d)\}$ spans V , a subspace of $\mathfrak{R}^3$. Determine all the values of c and d for which
- i) Dimension V is 1
- ii) Dimension V is 2 (10 marks)

QUESTION FIVE (20 MARKS)

- (a) Let $T: \mathfrak{R}^3 \rightarrow \mathfrak{R}^2$ be the linear transformation defined by

$$T(x_1, x_2, x_3) = (-x_1, 2x_2 - x_3)$$

- i) Show that $\mathbf{y} = (0, -1, -2)$ is in the kernel of T . (3 marks)

- ii) Evaluate the matrix of T with respect to the standard basis vectors in $\mathfrak{R}^3$ and $\mathfrak{R}^2$ respectively. (7 marks)
- (b) The total cost of 6 oranges, 2 bananas and 5 apples is sh. 210. This cost is three and a half times the total cost of 3 oranges, 1 banana and 1 apple. To buy 3 bananas and 2 apples of the same type, Betty requires to pay sh. 90. If the variables x , y and z represent the types of fruits respectively,
- i) Write the system of equations from the given information. (3 marks)
 - ii) Calculate the total cost of 9 oranges, 10 bananas and 8 apples. (7 marks)

