LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

1ST SEMESTER 2022/2023 ACADEMIC YEAR

FIRST YEAR EXAMINATION FOR THE DEGREES OF BACHELOR OF SCIENCE (STATISTICS), BACHELOR OF SCIENCE (GENERAL)

MATH 121: CALCULUS I

STREAM: R ***TIME: 2 HRS***

DAY: MONDAY [8.30AM – 10.30AM] ***DATE: 05/12/2022***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS TO CANDIDATES

- Answer Question ONE (COMPULSORY) and ANY TWO Questions
- Do not write on the question paper.
- LAIKIPIA UNIVERSITY observes ZERO tolerance to examination cheating.

QUESTION ONE (30 MARKS)

(a) . Find the following limits

(i). $\lim_{x\to -1} \frac{2x^2 - x - 2}{x+1}$ (3 marks)

(ii). $\lim_{x\to 0} \frac{\sqrt{3+x} - \sqrt{x}}{x}$ (3 marks)

(b). (i). Define continuity of a function (1 mark)

(ii). Find the constant a so that the function is continuous on the entire real line $f(x) = \begin{cases} x^3, x \leq 2 \\ ax^2, x \angle 2 \end{cases}$ (3 marks)

(iii). State the three conditions that must be satisfied for $f(x)$ to be continuous

(3 marks)

(c). Find the derivative of $y = \frac{t^2+1}{t}$ by first principles (4 marks)

(d). Find the value of the derivative at the indicated point $f(x) = 3x\left(x^2 - \frac{2}{x}\right), (2,18)$

(3 marks)

(e). Find $\frac{dy}{dx}$ of the following functions

(i). $y = x^2(2x-1)$ (3 marks)

(ii). $y = \dfrac{x^4 - 5}{x^2 + 3}$ (3 marks)

(f). The sum of two positive numbers is 16, find the numbers if the product of the squares is to be a maximum (4 marks)

QUESTION TWO (20 MARKS)

(a) . If $y = e^x \sin x$, show that $\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} + 2y = 0$ (6 marks)

(b) . Given that $x = t + \dfrac{1}{t}, y = t - \dfrac{1}{t}$ find $\dfrac{d^2y}{dx^2}$ (6 marks)

(c) . The total profit y(in shillings) of a company from the manufacture and sale of x bottles is given by, $y = -\dfrac{x^2}{400} + 2x - 80$

(i) . How many bottles must the company sell to achieve maximum profit (4 mark)

(ii). What is the profit per bottle when this maximum is achieved (4 marks)

QUESTION THREE (20 MARKS)

(a) . Find the derivatives of the following functions

(i). $f(x) = \dfrac{\sec x}{1 + \tan x}$ (3 marks)

(ii). $y = \sqrt{2x + 3}$ (3 marks)

(iii). $y = \ln(2x^3)$ (3 marks)

(b) . Given $y = x^{\frac{1}{2}}$ find an approximate value of $\sqrt{4.01}$ (4 marks)

(c) . P is the point (4,8) on the curve $y = x^2 - 5x + 12$

(i). find the equation of the tangent to the curve at P (4 marks)

(ii). Find also the equation of the normal to the curve at P (3 marks)

QUESTION FOUR (20 MARKS)

(a). (a) Find y'' given that

(i) $x = ye^x$ (4marks)

(ii). $xy^4 + x^2y = x + 3y$ (4 marks)

(iii). $y = x^x$ (4 marks)

(b). Sketch the curve $y = 3x^2 - 2x^3$ (8 marks)

QUESTION FIVE (20 MARKS)

(a). Find the constants A, B and C such that the function $y = Ax^2 + Bx + C$ satisfies the differential equation $y'' + y' - 2y = x^2$ (6 marks)

(b). Use L'hopitals rule to evaluate the limit $\lim_{x \to -3} \frac{x^2 - 9}{x^2 + 2x - 3}$ (4 marks)

(c). Find the maximum and minimum values of the function (6 marks)

$$y = 2x^3 - 3x^2 - 36x + 10$$

(d). The value of a computer t years after purchase is $v(t) = 2000e^{-0.35t}$ shillings. At what rate is the computer's value falling after 3 years. (4 marks)