

UNIVERSITY EXAMINATIONS

1ST SEMESTER 2022/2023 ACADEMIC YEAR

SECOND YEAR EXAMINATION FOR THE DEGREE OF
BACHELOR OF SCIENCE (STATISTICS)

STAT 213: PRINCIPLES OF STATISTICAL INFERENCE

STREAM: R

TIME: 2 HRS

DAY: WED [11.30AM – 1.30PM]

DATE: 14/12/2022

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

PLEASE DO NOT OPEN UNTIL THE INVIGILATOR SAYS SO.

INSTRUCTIONS: Answer question ONE and any other TWO questions

QUESTION ONE (30 Marks)

a) Explain briefly the difference between the following terms as used in hypothesis testing:

- i) Null and alternative hypotheses
- ii) Type I and type II errors
- iii) One tailed and two tailed tests

(6 Marks)

b) If S_1^2 and S_2^2 represent the variances of independent random samples of sizes $n_1 = 9$ and $n_2 = 11$, taken from normal populations with variances $\sigma_1^2 = 16$ and $\sigma_2^2 = 10$, respectively, find $P\left(0.6299 < \frac{S_1^2}{S_2^2} < 6.16\right)$.

(5 Marks)

c) The mean score of students on an aptitude test, at a certain college, is 72 points and standard deviation of 8 points. What is the probability that two groups of students consisting of 28 and 36 students, respectively, will differ in their mean scores by more than 4 points?

(5 Marks)

d) A pharmaceutical company makes tranquilizers. It is assumed that the distribution for the length of time they last is approximately normal. Researchers in a hospital used the drug on a random sample of twelve patients. The effective period of the tranquilizer for each patient (in hours) was as follows: 2.0, 2.7, 3.2, 2.8, 3.1, 2.6, 2.3, 2.2, 2.8, 2.4, 2.5 and 2.4. Find a 99% confidence interval for the population mean time. Interpret it.

(6 Marks)

e) A magazine claims that 25% of its readers are college students. Do we have a reason to doubt this claim if, in a random sample of 200 readers it is found that 42 are college students?

(5 Marks)

QUESTION TWO (20 Marks)

a) Suppose that 2% of all cell phone connections by a certain provider are dropped. Find the probability that in a random sample of 1,500 calls at most 40 will be dropped

(5 Marks)

b) Find the probability that a random sample of 21 observations, from a normal population with variance $\sigma^2 = 8$ will have a variance s^2 between 3.304 and 7.736.

(5 Marks)

- c) An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed, with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that the average life of a random sample of 16 bulbs will be
- Less than 775 hours
 - More than 822 hours
 - More than 809 hours but less than 828 hours

(10 Marks)

QUESTION THREE (20 Marks)

The closing prices of two common stocks were recorded for a period of 10 days. The means and variances are $\bar{x}_1 = 40.33$, $s_1^2 = 1.54$, $\bar{x}_2 = 42.54$ and $s_2^2 = 2.96$.

- Find a 95% confidence interval for the difference in average closing prices for the two stocks. Assume that populations are normally distributed with equal variances
- Use the above confidence interval to determine whether there was a significant difference in average closing prices of the two stocks.
- Can we conclude that the average closing price of the second stock was greater than that of the first stock?
- Find a 95% confidence interval for the ratio of the population variances. Was the assumption of equal variances in part a) justified?

(8 Marks)

(3 Marks)

(3 Marks)

(6 Marks)

QUESTION FOUR (20 Marks)

- The life in hours of a battery is known to be approximately normally distributed, with a standard deviation of 1.25 hours. A random sample of 10 batteries has a mean life of $\bar{x} = 42.65$ hours.
 - Suppose your research objective is to show that the mean life exceeds 40 hours. State the null and alternative hypotheses for the test.
 - Is there evidence to support the claim that the mean life exceeds 40 hours? Use $\alpha = 0.05$.

(8 Marks)



- b) A certain change in a process for manufacture of component parts is being considered. Samples are taken using both the existing and the new procedure so as to determine if the new process results in an improvement. Suppose 75 of 1500 items from the existing procedure were found to be defective and 80 of 2000 items from the new procedure were found to be defective. Do these data present sufficient evidence to indicate that the new process results in a significant improvement? (12 Marks)

QUESTION FIVE (20 Marks)

- a) The number of books borrowed from a library during a certain week were 518 on Monday, 431 on Tuesday, 485 on Wednesday, 443 on Thursday and 523 on Friday. Is there any evidence that the number of books borrowed varies between the five days of the week? Use a 1% level of significance. (5 Marks)
- b) The table below summarizes the times taken, in seconds, by 250 nine-year-old children to tie their shoe laces.

Time (sec.)	10 - 19	20 - 29	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79
Frequency	9	16	51	72	67	11	4

Does the assumption of the normal distribution with a mean 45 and standard deviation 10 seem appropriate as a probability model for this data? Use $\alpha = 0.05$.

(9 Marks)

- c) In an experiment to study the dependence of hypertension on smoking habits, the following data were taken on 180 individuals:

	Non-smokers	Moderate Smokers	Heavy
Smokers			
Hypertension	21	36	30
No hypertension	48	26	19

Test the hypothesis, at 0.05 level of significance, that the presence or absence of hypertension is independent of smoking habits. (6 Marks)