LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST SEMESTER 2025/2026 ACADEMIC YEAR

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (GENERAL)

MATH 212: VECTOR ANALYSIS

STREAM: R ***TIME: 2 HRS***

DAY: WEDNESDAY [11.30 – 1.30 P.M] ***DATE: 28/01/2026***

THIS QUESTION PAPER CONSISTS OF THREE (3) PAGES

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INSTRUCTION: ANSWER QUESTION ONE ANY OTHER TWO

QUESTION ONE (30 MARKS)

a) Distinguish between a solenoidal and an Irrotational vector **(2 marks)**

b) Given the vectors $\vec{A}$=8$\hat{\imath}$+2$\hat{\jmath}$+4$\hat{k}$ and $\vec{B}$=-3$\hat{\imath}$+4$\hat{\jmath}$-5$\hat{k}$: find

i) The magnitude of the vector $\vec{A}\times\vec{B}$ **(4 marks)**

ii) Directional cosines of the vector $\vec{B}$ **(2 marks)**

iii) Unit vector in the direction of $\vec{B}$ **(2 marks)**

iv) $(\vec{A}+2\vec{B})\cdot(-5\vec{A}-\vec{B})$ **(3 marks)**

c) Given $\vec{r}$ to be a position vector ,show that $(\vec{f}\cdot\nabla)\vec{r}=\vec{f}$ **(3 marks)**

d) A particle moves along a curve whose parametric equations are;

$x=e^{-2t}$,$y=\cos 3t$,$z=2\sin 3t$, where t is the time determine

i) Its velocity and acceleration at any time **(4 marks)**

ii) The magnitude of the velocity and acceleration at $t=0$ **(3 marks)**

e) Evaluate

i) $\int_0^2\int_1^3 xy^2\,dydx$ **(3 marks)**

ii) $\iiint_B x^3yz^2\,dv$ given that $\{B=x,y,z \mid 0\le x\le 2, -2\le y\le 3, 0\le z\le 1\}$

(4 marks)

QUESTION TWO (20 MARKS)

a) Define

i) a cylindrical co-ordinate system **(2 marks)**

ii) Convert the Cartesian coordinates for $(4,-5,2)$ into Cylindrical coordinates. **(3 marks)**

iii) Use cylindrical co-ordinates system to evaluate $\iiint_v x\,dv$ where v is the cylinder

bounded by $x^2+y^2=4, z=0, z=3$ **(5 marks)**

b) Given the vectors $\vec{A}=\hat{\imath}-3\hat{\jmath}+\hat{k}$ and $\vec{B}=3\hat{\imath}+4\hat{\jmath}-\hat{k}$. Find;

i) Angle between $\vec{A}\ and \vec{B}$ **(3 marks)**

ii) $(\overrightarrow{B}\cdot\overrightarrow{A})$ **(3 marks)**

iii) $|(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})|$ **(4 marks)**

QUESTION THREE (20 MARKS)

a) Proof that $\frac{d}{du}(\vec{A}\times\vec{B})=\vec{A}\times\frac{d\vec{B}}{du}+\frac{d\vec{A}}{du}\times\vec{B}$ **(4 marks)**

b) Given $\vec{A} = \boldsymbol{sin}\, xy\hat{\imath} + (3y - 2x)\hat{\jmath} - (3x + 2y)\hat{k}$;Evaluate

i) $\frac{\partial^2 \vec{A}}{\partial x \partial y}$ **(3 marks)**

ii) $\frac{\partial^2 \vec{A}}{\partial y\, \partial y}$ **(3 marks)**

c) i) State the Green theorem **(2 marks)**

iii) Verify the greens theorem in the integral $\oint(xy + y^2)\, dx + (x^2)dy$ where c is the closed curve of the region bounded by $y = x$ and $y = x^2$ **(6 marks)**

d) List two vector quantities **(2 marks)**

QUESTION FOUR (20 MARKS)

a) Given that $\vec{F} = xz^3\hat{\imath} - 2xyz\hat{\jmath} + xz\hat{k}$,find

i) Div $\vec{F}$ at the point(1,2,0) **(4 marks)**

ii) Curl of $\vec{F}$ at the point (-4,0,-2) **(4 marks)**

b) What are coplanar vectors **(2 marks)**

c) i) State gauss' Divergence Theorem **(2 marks)**

ii) Evaluate $\oiint \vec{A}.\vec{n}\, ds$ over the region bounded by $x = 0, y = 0, z = 0, x = a, y = bz = c$ where $\vec{A} = (x^2 + yz)\hat{\imath} + (y^2 - zx)\hat{\imath} + (z^2 - xy)\hat{k}$ using divergence theorem **(4 marks)**

d) Determine the value of ∇Q at the points $(1,\frac{\pi}{2},1)$ given that $Q = z\, sin(xy - \frac{\pi}{2})$ **(4 marks)**

QUESTION FIVE (20 MARKS)

a) Given the curve $x = t^3, y = 4t^2, z = 2t^2 - 6t$.Find ;

i) The unit tangent vector to any point on the curve **(4 marks)**

ii) Determine the unit tangent at $t = -3$ **(2 marks)**

b) Given $u = xy^2z$ and $\vec{r} = xz\hat{\imath} - xy\hat{\jmath} + yz^2\hat{k}$.Find

i) $\frac{\partial^2}{\partial x \partial z}(u\vec{r})$ **(3 marks)**

ii) $\frac{\partial^3}{\partial x^2 \partial z}(u\vec{r})$ **(3 marks)**

c) Stake stokes theorem **(2 marks)**

d) Verify stokes theorem given $\vec{A} = (x + y)\hat{\imath} + (2y - x)\hat{\jmath} + z^2\hat{k}$ and S is the upper surface of the sphere $x^2 + y^2 + z^2 = 1$ **(6 marks)**