LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE DEGREES OF BACHELOR OF EDUCATION (SCIENCE), BACHELOR OF EDUCATION (ARTS), & BACHELOR OF SCIENCE (STATISTICS)

MATH 315: COMPLEX ANALYSIS 1

STREAM: R ***TIME: 2 HRS***

DAY: MONDAY [11.30A.M – 1.30P.M] ***DATE: 15/04/2024***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS

Answer **ALL** questions in section **A** and any **TWO** in section **B**

SECTION A

QUESTION ONE (30 MARKS)

a) Express the following complex numbers in polar form

(i) $z = 2 + 2\sqrt{3}i$ **(2 Marks)**

(ii) $z = \left(i + \sqrt{2}\right)^{\frac{1}{3}}$ **(2 Marks)**

b) Find the fifth root of unity. **(3 Marks)**

c) Evaluate $\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3x - y)dy$ along the straight lines from $(0,3)$ to $(2,3)$ and then from $(2,3)$ to $(2,4)$. **(6 Marks)**

d) Determine the Laurent series for $f(z) = \dfrac{1}{(z+3)(z+1)}$ valid for $0 \prec z+1 \prec z$. **(4 Marks)**

e) Expand $f(z) = \sin z$ in Taylors series about $z = \dfrac{\pi}{4}$. **(5 Marks)**

f) Determine if the function $f(z) = z^2 + z + 2$ is analytic. **(4 Marks)**

g) Find the cube root of the complex number $z = -1 + i$. **(4 Marks)**

SECTION B

QUESTION TWO (20 MARKS)

a) Evaluate $\int_{(0,1)}^{(2,5)} (3x + y)dx + (2y - x)dy$ along

(i) The curve $y = x^2 + 1$. **(2 Marks)**

(ii) The straight line from $(0,1)$ to $(0,5)$ and then from $(0,5)$ to $(2,5)$. **(5 Marks)**

b) By expressing e^{iz} and e^{-iz} in terms of $\sin(z)$ and $\cos(z)$ using Euler's formula, show that

$\cos(z)=\frac{e^{iz}+e^{-iz}}{2}$ and $\sin(z)=\frac{e^{iz}-e^{-iz}}{2i}$. **(5 Marks)**

c) Simplify the expression $\frac{[\cos(\theta)+i\sin(\theta)][\cos(3\phi)+i\sin(3\phi)]}{[\cos(2\theta)-i\sin(2\theta)][\cos(\phi)-i\sin(\phi)]}$. **(3 Marks)**

d) Given that $a+bi=\frac{1}{5-3i}$, determine the values of a and b **(5 Marks)**

QUESTION THREE (20 MARKS)

a) Verify Cauchy-Goursat's theorem for the expression $f(z)=z^2+z+1$ where C is a rectangle with vertices $A(1,1), B(1,3), C(4,3)$ and $D(4,1)$. **(10 Marks)**

b) State the Laurent theorem. Hence find Laurent series about the indicated singularities for each of the following functions; **(2 Marks)**

(i) $\frac{e^{2z}}{(z-1)^3}$, $z=1$ **(4 Marks)**

(ii) $\frac{1}{z-3}$, $|z|\prec 3$ **(4 Marks)**

QUESTION FOUR (20 MARKS)

a) State Cauchy-Riemann equations. **(2 Marks)**

b) Prove that $u(x,y)=2+3x-y+x^2-y^2-4xy$ is harmonic. Hence find its conjugate harmonic function v such that $f(z)=u+iv$ is analytic. **(5 Marks)**

c) State the Cauchy's integral formula. Hence evaluate

(i) $\oint_c \frac{e^{-iz}}{z^4}dz$ where C is the circle $|z|=1$. **(5 Marks)**

(ii) $\oint_c \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)(z-2)}dz$ where C is the circle $|z|=3$. **(5 Marks)**

d) If $z_1=4+2i$ and $z_2=2-3i$ determine $\arg(\frac{z_2}{z_1})$ **(3 Marks)**

QUESTION FIVE (20 MARKS)

a) State the Residue theorem. Hence evaluate; **(2 Marks)**

(i) $\oint_c \frac{2z+3}{z-1}dz$ where C is the circle $|z|=3$. **(4 Marks)**

(ii) $\oint_c \frac{e^z}{\left(z^2+\pi^2\right)^2}dz$ where C is the circle $|z|=4$. **(6 Marks)**

b) Prove that $\sum_{n=1}^{\infty}\frac{z^n}{n(n+1)}$ converges absolutely for $|z|\le 1$. **(4 Marks)**

c) Find the region of convergence of the series $\sum_{n=1}^{\infty}\frac{(z+2)^{n-1}}{(n+1)^3 4^n}$. **(4 Marks)**