



# UNIVERSITY EXAMINATIONS

**FIRST SEMESTER 2025/2026 ACADEMIC YEAR**

**FOURTH YEAR EXAMINATION FOR THE DEGREES OF  
BACHELOR OF SCIENCE (STATISTICS) AND BACHELOR  
OF (ECONS & STATISTICS)**

**STAT 413: NON - PARAMETRIC INFERENCE**

***STREAM: R***

***TIME: 2 HRS***

***DAY: WEDNESDAY [11.30 – 13.30 P.M]***

***DATE: 04/02/2026***

**THIS QUESTION PAPER CONSISTS OF SEVEN (7) PAGES**

**PLEASE DO NOT OPEN UNTIL THE INVIGILATOR SAYS SO.**

**QUESTION ONE [30 MARKS]**

- a) Explain the term non-parametric test [2 Marks]
- b) State any three advantages of non-parametric tests [3 Marks]
- c) What is meant by ranking data? Perform ranking given the scores: 90, 85, 95, 70, 75, 80, 55, 90 [3 Marks]
- d) Wilcoxon signed-rank test has been considered as advancement of sign test. Explain why that is the case. [3 Marks]
- e) Distinguish between simple and composite hypothesis [3 Marks]
- f) What is a contingency table? Use an appropriate illustration to explain. [3 Marks]
- g) For every non-parametric test there is an equivalent for each parametric general type of test. State three broad categories that these tests fall into. [3 Marks]
- h) Consider the table below on non-parametric statistical procedures as used in research and fill in the missing entries (?) appropriately [5 Marks]

Type of analysis	Non parametric test	Parametric equivalent
Comparing two related samples		t-test for independent samples
?	Mann–Whitney <i>U</i> -test and Kolmogorov–Smirnov two-sample test	?
Comparing three or more unrelated samples	?	?

- i) Suppose in a study comparing two teaching methods for reading recovery gave the following data  

Pull-out (P)	48	40	39	50	21	
Small group (S)	14	18	28	10	12	17

 Compute Mann-Whitney U Statistic [5 Marks]

**QUESTION TWO [20 MARKS]**

- a) When is a permutation test used? Give its assumptions. [5 Marks]
- b) Give test statistic for Wilcoxon signed rank test and give its three assumptions. [4 Marks]
- c) Assume a physical education teacher conducted an action research project to examine a strength and conditioning program using six participants, she measures the number of curl ups they could do in 1 min. She also measured their performance before and after the programs. Write down R codes that were used to generate the outputs on the right based on the above research. [4 Marks]

	before	after
1	12	9
2	15	5
3	8	10
4	11	3
5	9	4
6	17	2

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Wilcoxon signed rank exact test
data: before and after
V = 20, p-value = 0.03125
alternative hypothesis: true location shift is greater than 0
    
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- d) Assume that in a controlled clinical trial, participants were randomly assigned to two groups aspirin and placebo where the first six values are treatments while the rest are controls.  
 3120, 2104, 1664, 2481, 2603, 2301, 1620, 1743, 1397, 1503, 2339, 2090, 2231, 1225  
 Perform a permutation test. [7 Marks]

**QUESTION THREE [20 MARKS]**

a) Assume a coin was tossed twenty times and sequence of heads (H) and tails (T) is as given below.

THTTTHTHTHHTHTTTHHTTH

- i) Count the number of runs [2 Marks]
- ii) Test whether the Heads and the Tails occur in random order  $[\alpha = 0.05]$  [3 Marks]

b) Suppose a researcher investigated how physical attraction influences perception among others of a person’s effectiveness with difficult tasks. The photographs of eighteen people were shown to a focus group and asked to classify the photos into three groups: very attractive, average, and very unattractive and then R output was generated from their results.

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Kruskal-Wallis rank sum test

data: list(very_attractive, average, very_unattractive)
Kruskal-Wallis chi-squared = 6.8568, df = 2, p-value = 0.03244
    
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- i) Write down with explanations R commands that were used [4 Marks]
  - ii) Give an appropriate interpretation [3 Marks]
- c) Suppose an experiment was carried out to determine gas cutting of steel for use in off - shore structures where twelve test cars were prepared. Each plate was cut using both system one and system two and, in each case, the maximum Vickers hardness near the cut edge was measured and results were as shown below.

Car	1	2	3	4	5	6	7	8	9	10	11	12
System 1	86	71	77	68	91	72	77	91	70	71	88	87
System 2	88	77	76	64	96	72	65	90	65	80	81	72

Use Wilcoxon signed-rank test to determine whether Vickers hardness differs significantly  $[\alpha = 0.05, T = 1.6449]$  [8 Marks]

**QUESTION FOUR [20 MARKS]**

a) Suppose springs used in the lids of portable CD players are subjected to testing by repeated flexing until they fail. Those times marked with \* in table below indicate cases where the experiment was stopped before the spring failed, the times in minutes to failure of sixteen springs are given below.

*48.0	41.2	1.2	*48.0	*48.0	0.7	0.2	12.2
1.9	0	42.6	*48.0	15.7	*48.0	4.3	24.2

Perform a sign test.  $[\alpha = 0.05]$  [10 Marks]

b) Consider the following data that was taken from a study to determine nurses’ attitude of readiness to care for patients .

Participant	1	2	3	4	5	6	7	8	9	10	11
Gender	M	M	M	M	M	F	F	F	F	F	F
Attitude	+	+	-	+	+	-	-	-	-	+	-

Determine if there is a difference in attitude between male and female toward their preparation to care for patients. [ $\alpha = 0.05$ ] **[10 Marks]**

**QUESTION FIVE [20 MARKS]**

- a) When is fisher’s exact test considered appropriate to use? **[3 Marks]**
- b) Suppose the following data was from a follow up study on breast cancer in country A, perform a test of outliers to detect any outliers and make necessary comment on your results. **[7 Marks]**

5.4 22.2 29.8 8.9 6.2 7.2 3.7 2.8 12.7 6.9 3.1

- c) Consider the data on lifespan of groups of rats.

18.2, 21.4, 22.6, 17.4, 17.6, 16.7, 17.1, 21.4, 20.1, 17.9, 16.8, 23.1

Conduct Kolmogorov Smirnov test to determine whether this data was drawn from a normal population at  $\alpha = 0.01$  **[10 Marks]**

**TABLE B.3 Critical Values for the Wilcoxon Signed Rank Test Statistics  $T$ .**

$n$	$\alpha_{\text{two-tailed}} \leq 0.10$	$\alpha_{\text{two-tailed}} \leq 0.05$	$\alpha_{\text{two-tailed}} \leq 0.02$	$\alpha_{\text{two-tailed}} \leq 0.01$
	$\alpha_{\text{one-tailed}} \leq 0.05$	$\alpha_{\text{one-tailed}} \leq 0.025$	$\alpha_{\text{one-tailed}} \leq 0.01$	$\alpha_{\text{one-tailed}} \leq 0.005$
5	0			
6	2	0		
7	3	2	0	
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37
21	67	58	49	42
22	75	65	55	48
23	83	73	62	54
24	91	81	69	61
25	100	89	76	68
26	110	98	84	75
27	119	107	92	83
28	130	116	101	91
29	140	126	110	100
30	151	137	120	109

**TABLE A13 Quantiles of the Kolmogorov Test Statistic<sup>a</sup>**

One-Sided Test					Two-Sided Test						
$p = 0.90$					$p = 0.90$						
$p = 0.95$					$p = 0.95$						
$p = 0.975$					$p = 0.975$						
$p = 0.99$					$p = 0.99$						
$p = 0.995$					$p = 0.995$						
$p = 0.80$					$p = 0.80$						
$n = 1$	0.900	0.950	0.975	0.990	0.995	$n = 21$	0.226	0.259	0.287	0.321	0.344
2	0.684	0.776	0.842	0.900	0.929	22	0.221	0.253	0.281	0.314	0.337
3	0.565	0.636	0.708	0.785	0.829	23	0.216	0.247	0.275	0.307	0.330
4	0.493	0.565	0.624	0.689	0.734	24	0.212	0.242	0.269	0.301	0.323
5	0.447	0.509	0.563	0.627	0.669	25	0.208	0.238	0.264	0.295	0.317
6	0.410	0.468	0.519	0.577	0.617	26	0.204	0.233	0.259	0.290	0.311
7	0.381	0.436	0.483	0.538	0.576	27	0.200	0.229	0.254	0.284	0.305
8	0.358	0.410	0.454	0.507	0.542	28	0.197	0.225	0.250	0.279	0.300
9	0.339	0.387	0.430	0.480	0.513	29	0.193	0.221	0.246	0.275	0.295
10	0.323	0.369	0.409	0.457	0.489	30	0.190	0.218	0.242	0.270	0.290
11	0.308	0.352	0.391	0.437	0.468	31	0.187	0.214	0.238	0.266	0.285
12	0.296	0.338	0.375	0.419	0.449	32	0.184	0.211	0.234	0.262	0.281
13	0.285	0.325	0.361	0.404	0.432	33	0.182	0.208	0.231	0.258	0.277
14	0.275	0.314	0.349	0.390	0.418	34	0.179	0.205	0.227	0.254	0.273
15	0.266	0.304	0.338	0.377	0.404	35	0.177	0.202	0.224	0.251	0.269
16	0.258	0.295	0.327	0.366	0.392	36	0.174	0.199	0.221	0.247	0.265
17	0.250	0.286	0.318	0.355	0.381	37	0.172	0.196	0.218	0.244	0.262
18	0.244	0.279	0.309	0.346	0.371	38	0.170	0.194	0.215	0.241	0.258
19	0.237	0.271	0.301	0.337	0.361	39	0.168	0.191	0.213	0.238	0.255
20	0.232	0.265	0.294	0.329	0.352	40	0.165	0.189	0.210	0.235	0.252
Approximation for $n > 40$							$\frac{1.07}{\sqrt{n}}$	$\frac{1.22}{\sqrt{n}}$	$\frac{1.36}{\sqrt{n}}$	$\frac{1.52}{\sqrt{n}}$	$\frac{1.63}{\sqrt{n}}$

**TABLE A12** Quantiles of the Wilcoxon Signed Ranks Test Statistic

	$W_{0.005}$	$W_{0.01}$	$W_{0.025}$	$W_{0.05}$	$W_{0.10}$	$W_{0.20}$	$W_{0.30}$	$W_{0.40}$	$W_{0.50}$	$\frac{n(n+1)}{2}$
$n = 4$	0	0	0	0	1	3	3	4	5	10
5	0	0	0	1	3	4	5	6	7.5	15
6	0	0	1	3	4	6	8	9	10.5	21
7	0	1	3	4	6	9	11	12	14	28
8	1	2	4	6	9	12	14	16	18	36
9	2	4	6	9	11	15	18	20	22.5	45
10	4	6	9	11	15	19	22	25	27.5	55
11	6	8	11	14	18	23	27	30	33	66
12	8	10	14	18	22	28	32	36	39	78
13	10	13	18	22	27	33	38	42	45.5	91
14	13	16	22	26	32	39	44	48	52.5	105
15	16	20	26	31	37	45	51	55	60	120
16	20	24	30	36	43	51	58	63	68	136
17	24	28	35	42	49	58	65	71	76.5	153
18	28	33	41	48	56	66	73	80	85.5	171
19	33	38	47	54	63	74	82	89	95	190
20	38	44	53	61	70	83	91	98	105	210
21	44	50	59	68	78	91	100	108	115.5	231
22	49	56	67	76	87	100	110	119	126.5	253
23	55	63	74	84	95	110	120	130	138	276
24	62	70	82	92	105	120	131	141	150	300
25	69	77	90	101	114	131	143	153	162.5	325
26	76	85	99	111	125	142	155	165	175.5	351
27	84	94	108	120	135	154	167	178	189	378
28	92	102	117	131	146	166	180	192	203	406
29	101	111	127	141	158	178	193	206	217.5	435
30	110	121	138	152	170	191	207	220	232.5	465

**TABLE B.10 Critical Values for the Runs Test for Randomness.**

One-tailed alternative;  $\alpha = 0.05$ .

$n_1$	$n_2$										
	2	3	4	5	6	7	8	9	10	11	12
2	-	-	-	-	-	-	2	2	2	2	2
3	-	-	-	2	2	2	2	2	3	3	3
4	-	-	2	2	3	3	3	3	3	3	4
5	-	2	2	3	3	3	3	4	4	4	4
6	-	2	3	3	3	4	4	4	5	5	5
7	-	2	3	3	4	4	4	5	5	5	6
8	2	2	3	3	4	4	5	5	6	6	6
9	2	2	3	4	4	5	5	6	6	6	7
10	2	3	3	4	5	5	6	6	6	7	7
11	2	3	3	4	5	5	6	6	7	7	8
12	2	3	4	4	5	6	6	7	7	8	8