LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST SEMESTER 2025/2026 ACADEMIC YEAR

FOURTH YEAR EXAMINATION FOR THE DEGREES OF BACHELOR OF EDUCATION (SCIENCE/ARTS) AND BACHELOR OF SCIENCE (GENERAL)

MATH 411: PARTIAL DIFFERENTIAL EQUATIONS 1

STREAM: R ***TIME: 2 HRS***

DAY: THURSDAY [11.30 – 1.30 P.M] ***DATE: 29/01/2026***

THIS QUESTION PAPER CONSISTS OF THREE (3) PAGES

PLEASE DO NOT OPEN UNTIL THE INVIGILATOR SAYS SO.

Instructions: Answer question 1 (compulsory) and any other 2 (two) questions. There is no need to attempt any more questions.

QUESTION ONE (COMPULSORY, 30 MARKS)

a) Define the following terms as used in the field of differential equations

(i) Partial differential equation (vis-à-vis an ordinary differential equation) **(4 Marks)**

(ii) The *order* and *degree* of a partial differential equation (give an example for each)

(4 Marks)

b) Solve the following ODEs

(i) $\frac{dx}{2z-5y} = \frac{dy}{5x-z} = \frac{dz}{y-2x}$ **(4 Marks)**

(ii) $\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{xyz^2(x^2-y^2)}$ **(5 Marks)**

c) (i) Define a homogeneous function $\Phi(x, y, z)$. **(2 Marks)**

(ii) Prove that a homogeneous Pfaffian DE $P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz = 0$ can be reduced to the form $\frac{dx}{x} + A(u, v)du + B(u, v)dv = 0$ **(5 Marks)**

(iii) The Pfaffian differential equation $(y^2 + yz)dx + z(x \quad + z)dy + y(y - x)dz = 0$ is integrable. By examining its terms P, Q and R, prove that it is also homogeneous, and state its degree **(6 Marks)**

QUESTION TWO (OPTIONAL, 20 MARKS)

a) Solve completely the equation given in Q1(c) (iii) above **(10 Marks)**

b) Use the Charpit method to solve the PDE $\left(\frac{dz}{dx}\right)^2 x + \left(\frac{dz}{dx}\right)^2 y = z$ **(10 Marks)**

QUESTION THREE (OPTIONAL, 20 MARKS)

a) Use the Jacobi method to solve $z^2 = pqxy$ **(12 Marks)**

b) Solve the following equations

(i) $\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{z}$

(ii) $\frac{dx}{6(y-z)} = \frac{2dy}{3(z-x)} = \frac{3dz}{2(x-y)}$ **(8 Marks)**

QUESTION FOUR (OPTIONAL, 20 MARKS)

a) Prove that the surfaces $x^2 + y^2 + z^2 = 30$ and $xy = 1$ intersect orthogonally at the point $P(1, -1, 0)$ **(6 Marks)**

b) A curve is given parametrically as $\overline{r} = 6sint\,\overline{\imath} + 4cos3t\overline{\jmath} + 2sin5t\overline{k}$. At the point corresponding to $t = \frac{\pi}{6}$ obtain the equations of its tangent line and normal plane. **(8 Marks)**

c) Prove that if the Pfaffian DE $Pdx + Qdy + Rdz$ is separable in one variable, (let us say z) follows that the part not containing z is automatically exact. **(6 Marks)**

QUESTION FIVE (OPTIONAL, 20 MARKS)

a) (i) Suppose $z = f(x, y)$ are surfaces orthogonal to a system of surfaces $F(x, y, z) = c$. Prove that the consequence is $\frac{dx}{\frac{\partial f}{\partial x}} = \frac{dy}{\frac{\partial f}{\partial y}} = \frac{dz}{\frac{\partial f}{\partial z}}$ **(4 Marks)**

(ii) Use these results to find the system of surfaces orthogonal to $f(x, y, z) = \frac{z(x+y)}{3z+1}$ **(5 Marks)**

b) (i) When is the equation $F(x, y, z, p, q) = 0$ *said to be separable*? **(1 Mark)**

(ii) By forming Charpit's equations for a separable equation, show that its solution is $f(x, p) = a\ (const)$. **(4 Marks)**

(iii) Use this fact to find the complete integral of the PDE $\left(\frac{dz}{dx}\right)^2 y(1 + x^2) = \frac{dz}{dx}x^2$ **(6 Marks)**