LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST SEMESTER 2025/2026 ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE DEGREES OF BACHELOR OF SCIENCE (GENERAL) AND BACHELOR OF SCIENCE (STATISTICS)

MATH 312: REAL ANALYSIS I

STREAM: R ***TIME: 2 HRS***

DAY: WEDNESDAY [11.30 – 13.30 P.M] ***DATE: 28/01/2026***

THIS QUESTION PAPER CONSISTS OF THREE (3) PAGES

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INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

(a) Define $f:\Re \mapsto \Re$ by $f(x)=\dfrac{x^3}{1+x^2}$. Show that f is continuous on $\Re$. Is f uniformly continuous on $\Re$?. **(9 Marks)**

(b) Show that $\sum_{n=1}^{\infty}\dfrac{1}{(n+1)(n+2)}=1$ **(6 Marks)**

(c) In calculus class we learn that the minimum or maximum of a continuous function $f:[a,b]\to\Re$ will be found at either

(i) A point of (a,b) where $f^1=0$.

(ii) A point of (a,b), where f is not differentiable.

(iii) One of the end point a,b.

Give a justification of this statement. **(5 Marks)**

(d) Let A *and* B be subsets of a universal set, let $A=\{1,2,4,5,6,12,13\}$ and $B=\{3,5,6,7,8,9,10\}$. Write down the elements of each of the following sets.

(i) $P=\{x\in U,\, x\in A \text{ or } x\in B\}$. **(2 Marks)**

(ii) $Q=\{x\in U,\, x\in A \text{ and } x\notin B\}$. **(2 Marks)**

(iii) $R=\{x\in U,\, x\in A \text{ and } x\in B\}$. **(2 Marks)**

(e) Suppose that $f(x)=\sum_{n=1}^{\infty}\dfrac{\sin nx}{n^3}$ and $g(x)=\sum_{n=1}^{\infty}\dfrac{\cos nx}{n^2}$. Prove that $f,g:\Re\to\Re$ are continuous . **(4 Marks)**

QUESTION TWO (20 MARKS)

(a) Consider the set of all rational numbers $S=\{x\in Q/\, x^2<2\}$. Does S have a least upper bound in the set of rational numbers, and if not, why?. **(4 Marks)**

(b) Use induction to prove that $\sum_{k=1}^{n}kx^{k-1}=\dfrac{1-(n+1)x^n+nx^{n+1}}{(1-x)^2}$ for all positive integers n. **(7 Marks)**

(c) Let $\Re$ be an ordered field. For $a,b,c\in\Re$. Show that

(i) For $a<b$, *then* $a+c<b+c$. **(2 Marks)**

(ii) For $a<b$ *and* $b<c$, *then* $a<c$. **(2 Marks)**

(d) Define absolute value function, hence show that for $a,b\in\Re$, $|a+b|\le|a|+|b|$. **(5 Marks)**

QUESTION THREE (20 MARKS)

(a) (i) Define a sequence $\{x_n\}$ of real numbers $\Re$. **(2 Marks)**

(ii) Define a convergence of $\{x_n\}$ of real numbers $\Re$. **(2 Marks)**

(iii) Evaluate the convergence if $\{x_n\}=\left\{1+\frac{1}{2n}\right\}$ *and* $\{y_n\}=\left\{2-\frac{1}{n^2}\right\}$ **(6 Marks)**

(b) Define a subsequence of real numbers. Hence Prove that a sequence $\{x_n\}$ of real numbers converges to L if and only if Every subsequence of $\{x_n\}$ converges to L. **(10 Marks)**

QUESTION FOUR (20 MARKS)

(a) A function f is defined by $f(x)=x^2+1,\ x\in\Re,\ x\geq 0$.

(i) Find an expression for $f^{-1}(x)$. **(2 Marks)**

(ii) State the domain and range of $f^{-1}(x)$. **(4 Marks)**

(b) The functions f *and* g are given by $f(x)=x^2,\ x\in\Re.\ g(x)=\frac{1}{x+2},\ x\in\Re,\ x\neq -2$.

(i) State the range of $f(x)$ **(1 Mark)**

(ii) Solve the equation $fg(x)=\frac{4}{9}$. **(4 Marks)**

(iii) Find, in its simplest form, an expression for $g^{-1}(x)$. **(3 Marks)**

(c) Prove that if $\{x_n\}$ is a convergent sequence of real numbers, then the sequence $\{x_n\}$ is bounded. **(6 Marks)**

QUESTION FIVE (20 MARKS)

(a) Which of the following functions are NOT everywhere continuous

$(i)\ f(x)=\frac{x^2-4}{x+2}.\ (ii)\ f(x)=(x+3)^4.\ (iii)\ f(x)=mx+b.\ (iv)\ f(x)=1066$ **(6 Marks)**

(b) Use functions in question (a) above. Which of the functions are NOT differentiable?.

(6 Marks)

(c) Find the derivatives of the following functions $(i)\ f(x)=1963.\ (ii)\ f(x)=+\infty$ **(2 Marks)**

(d) Is the function $f(x)=\begin{cases} 0 & if\ x=0 \\ x\sin\left(\frac{1}{x}\right) & if\ x\neq 0 \end{cases}$ continuous at point 0. **(6 Marks)**