LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

1^{ST} SEMESTER 2022/2023 ACADEMIC YEAR

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (GENERAL)

MATH 212: VECTOR ANALYSIS

STREAM: R ***TIME: 2 HRS***

DAY: WED [2.30PM – 4.30PM] ***DATE: 07/12/2022***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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<u>INSTRUCTIONS</u>: - Answer Question ONE and any other TWO questions

QUESTION ONE (30MKS)

(a) Classify the following quantities as scalars and vectors; (3 marks)

Weight, specific heat, volume, momentum, pressure and moment of a force

SCALARS	**VECTORS**

(b) Given the scalar field defined by If $\phi(x,y,z) = 3x^2y - 4y^2z + 5x^2z^2$ find $\vec{\nabla}\phi$ at the point (1,-2,0) (3 marks)

(c) Find a unit vector parallel to the resultant of the vectors; $\vec{r}_1 = 2\vec{i} + 4\vec{j} - 5\vec{k}$ and $\vec{r}_2 = 3\vec{i} - 2\vec{j} + \vec{k}$ (3 marks)

(d) Compute the volume of the parallelepiped whose edges are represented by $\vec{U} = 2\vec{i} - 3\vec{j} + 4\vec{k}$, $\vec{V} = \vec{i} + 2\vec{j} - \vec{k}$ and $\vec{C} = 3\vec{i} - \vec{j} + 2\vec{k}$ (3 marks)

(e) Find the equation for the tangent plane to the surface $z = x^2 + y^2$ at the point $(1,-1,2)$ (3 marks)

(f) Determine the velocity and acceleration of a particle which moves along the curve $x = 2\sin 3t, y = 2\cos 3t, z = 8t$ at any time $t > 0$. Determine the magnitude of the velocity and acceleration (4 marks)

(g) If $\vec{R}(u) = (u - u^2)\vec{i} + 2u^3\vec{j} - 3\vec{k}$ Find;

(i) $\int \vec{R}(u)du$ (3 marks)

(ii) $\int_1^2 \vec{R}(u)du$ (3 marks)

(h) Verify Green's theorem in the plane for (5 marks)

$$\oint_C (xy + y^2)dx + x^2dy$$

Where C is the closed curve of the region bounded by $y = x$ and $y = x^2$

The positive direction in traversing C is the counterclockwise direction

QUESTION TWO (20MKS)

(a) Distinguish between the divergence and curl of a differentiable vector field. (4 marks)

(b) Prove that the vectors $\vec{P} = 3\vec{i} + \vec{j} - 2\vec{k}$, $\vec{Q} = -\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{R} = 4\vec{i} - 2\vec{j} - 6\vec{k}$ can form the sides of a triangle. (4 marks)

(c) Find the projection of the vector $\vec{M} = -2\vec{i} + 5\vec{j} - 4\vec{k}$ on the vector $\vec{N} = \vec{i} - 3\vec{j} + 6\vec{k}$ (4 marks)

(d) The acceleration of a particle at any time $t \geq 0$ is given by;

$\vec{a} = \frac{d\vec{v}}{dt} = 12\cos 2t\vec{i} - 8\sin 2t\vec{j} + 16t\vec{k}$. If the velocity $\vec{v}$ and displacement $\vec{r}$ are zero at $t = 0$ Find $\vec{v}$ and $\vec{r}$ at any time (4 marks)

(e) Verify stokes' theorem for $\vec{A} = (y - z + 2)\vec{i} + (yz + 4)\vec{j} - xz\vec{k}$, where S is the surface of the cube $x = 0, y = 0, x = 2, y = 2, z = 2$ above the xy plane (4 marks)

QUESTION THREE (20MKS)

(a) Write short notes on the following;

- (i) Greens theorem (3 marks)
- (ii) Divergence theorem (3 marks)
- (iii) Stokes theorem (3 marks)

(b) Determine the values of λ so that $\vec{A} = \lambda\vec{i} - 2\vec{j} + \vec{k}$ and $\vec{B} = 2\lambda\vec{i} + \lambda\vec{j} - 4\vec{k}$ are perpendicular to each other. (3 marks)

(c) If $\vec{C} = 5t^2\vec{i} + t\vec{j} - t^3\vec{k}$, and $\vec{D} = \sin t\vec{i} - \cos t\vec{j}$, find;

(i) $\frac{d}{dt}(\vec{C} \bullet \vec{D})$ (4 marks)

(ii) $\frac{d}{dt}(\vec{C} \times \vec{D})$ (4 marks)

QUESTION FOUR (20MKS)

(a) Find the angle between the vectors $P = 3i - 4j + 5k$ and $Q = 2i + 3j - 4k$ (3 marks)

(b) Determine the direction cosines of the vector $\vec{T} = 4\vec{i} - 3\vec{j} + 7\vec{k}$ (3 marks)

(c) Find the directional derivative of $\phi(x, y, z) = x^2yz + 4xz^2$ at $(1,-2,-1)$ in the direction $2\vec{i} - \vec{j} - 2\vec{k}$ (3 marks)

(d) If $\vec{A} = 2x^2\vec{i} - 3yz\vec{j} + xz^2\vec{k}$ and $\phi(x,y,z) = 2z - x^3y$, find

(i) $\vec{A} \bullet \vec{\nabla}\varphi$ and (3 marks)

(ii) $\vec{A} \times \vec{\nabla}\varphi$ at the point $(1,-1,1)$ (3 marks)

(e) Verify the divergence theorem for $\vec{A} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ taken over the region bounded by $x^2 + y^2 = 4, z = 0$ and $z = 3$ (5 marks)

QUESTION FIVE (20MKS)

(a) If $\phi(x,y,z) = xy^2z$ and $\vec{F} = xz\vec{i} - xy^2\vec{j} + yz^2\vec{k}$, find $\frac{\partial^3}{\partial x^2 \partial z}(\phi\vec{F})$ at the point $(2,-1,1)$

(4 marks)

(b) Given that $\phi = \ln|\vec{r}|$ where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$ show that;

$$\vec{\nabla}\phi = \frac{\vec{r}}{r^2}$$ (4 marks)

(c) Find the unit normal to the surface $x^2y + 2xz = 4$ at the point $(2,-2,3)$ (4 marks)

(d) A vector $\vec{V}$ is said to be irrotational if Curl $\vec{V} = 0$. Find the constants α, β and γ so that $\vec{V} = (x + 2y + \alpha z)\vec{i} + (\beta x - 3y - z)\vec{j} + (4x + \gamma y + 2z)\vec{k}$ is irrotational (4 marks)

(a) Use Green's theorem to evaluate $\oint (y - \sin x)dx + \cos x dy$ where C is the triangle shown below; (4 marks)