LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

1ST SEMESTER 2022/2023 ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

MATH 318: CALCULUS III

STREAM: R ***TIME: 2 HRS***

DAY: TUESDAY [8.30AM – 10.30AM] ***DATE: 20/12/2022***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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QUESTION ONE (30 MARKS)

(a) Calculate the partial derivatives of the following;

i. $z = x^2 + 2xy$ (2 marks)

ii. $w = qr + \frac{pr}{q-p}$ (5 marks)

(b) Given an implicit function $(x^2 + y^2)^3 - 3(x^2 + y^2) + 1 = 0$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. (4 marks)

(c) Evaluate $\int_0^1 \int_0^x 2e^{x^2} \, dydx$ (3 marks)

(d) Find Taylors series for $f(x) = e^x$ about $x = 0$. (3 marks)

(e) List the first three terms of the following sequence $\left\{ \frac{(-1)^{n+1}}{2n + (-3)^n} \right\}_{n=2}^{\infty}$ (3 marks)

(f) Use geometric series to write the indefinite number $0.131313\ldots$as a rational number.

(4 marks)

(g) Find the equation of the tangent plane and the equation of the normal to the surface

$z = \frac{x^2}{2} - y^2$ at the point $(2,-1,1)$. (6 marks)

QUESTION TWO (20 MARKS)

(a) Evaluate the following limits:

i. $\lim_{n \to \infty} \sqrt[n]{2^{n+1}}$ (2 marks)

ii. $\lim_{n \to \infty} \left(\frac{1}{n} \right)^{\frac{-5}{n}}$ (2 marks)

(b) Determine and classify the critical points of $f(x, y) = xy - x^3 - y^2$. (8 marks)

(c) Use the method of Lagrange multipliers to find the minimum value of

$f(x, y) = x^2 + 4y^2 - 2x + 8y$ subject to the constraint $x + 2y = 7$. (8 marks)

QUESTION THREE (20 MARKS)

(a) Define the following terms;

(i) Infinite series. (1 mark)

(ii) Convergence of a sequence. (2 marks)

(b) Write the general term in each of the following series;

(i) $\frac{1}{3}+\frac{1}{8}+\frac{1}{15}+\frac{1}{24}+\ldots$ (2 marks)

(ii) $\frac{3}{4}+\frac{5}{4^2}+\frac{7}{4^3}+\ldots$ (2 marks)

(c) Use the method of differentials to approximate the value of $(1.03)^3(0.99)^2$. (4 marks)

(d) Evaluate $\iint e^{2x+3y}dxdy$ over the triangle bounded by the lines $x=0$, $y=0$ and $x+y=1$.

(4 marks)

(e) Use comparison test to examine the convergence of $\sum_{n-1}^{\infty}\frac{n^2+2}{n^4+5}$. (5 marks)

QUESTION FOUR (20 MARKS)

(a) Use ratio test to examine the convergence or divergence of;

(i) $\sum_{n=1}^{\infty}\frac{(-10)^n}{4n^{2n+1}}$ (4 marks)

(ii) $\sum_{n=1}^{\infty}\frac{1}{n!}$ (4 marks)

(b) Find the limits of the following sequences and confirm the convergence;

(i) $\left(\frac{2^n+1}{e^n-1}\right)$ (2 marks)

(ii) $\left(\frac{2-n^2}{3+n}\right)^n$ (3 marks)

(c) Expand $f(x)=\sin x$ using the Maclaurin series. (4 marks)

(d) Find the equation of the tangent line to the ellipse $x^2+2y^2=6$ at the point $(2,1)$.

(2 marks)

QUESTION FIVE (20 MARKS)

(a) Show that the equation $f(x,y) = x^2 + xy + y^2$ satisfies the Euler's Theorem.

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf(x,y)$$ (3 marks)

(b) Determine the total differential of $w = \sqrt{x^2 + y^2 + z^2}$ (4 marks)

(c) Find $\frac{du}{dt}$ if $u = xyz$ and $x = t^2 + 1$, $y = \ln t$, $z = \tan t$. (3 marks)

(d) A golf ball manufacturer, Pro-T has developed a profit model that depends on the number of x golf balls sold per month (measured in thousands), and the number of hours per month of advertising y, according to the

function $z = f(x,y) = 48x + 96y - x^2 - 2xy - 9y^2$,

where z is measured in thousands of dollars. The budgetary constraint function relating the cost of production of thousands of golf balls and advertising units is given by $20x + 4y = 216$. Find the values of x and y that maximize profit, and find the maximum profit. (10 marks)