LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

FOURTH YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (STATISTICS)

STAT 424: DESIGN AND ANALYSIS OF EXPERIMENTS II

STREAM: R ***TIME: 2 HRS***

DAY: MONDAY [8.30 A.M – 10.30 A.M] ***DATE: 08/04/2024***

THIS QUESTION PAPER CONSISTS OF FIVE (5) PAGES

PLEASE DO NOT OPEN UNTIL THE INVIGILATOR SAYS SO

INSTRUCTIONS

(i) Answer question one and any other two questions

(ii) Show all the workings clearly

(iii) Do not write on the question paper

QUESTION ONE (30 MARKS)

a. State three advantages of factorial designs **(3 Marks)**

b. In a balanced incomplete block design, explain what is meant by the two words: Balanced and incomplete. **(3 Marks)**

c. Define a treatment contrast and show when two treatment contrasts are orthogonal. **(4 Marks)**

d. Consider the model of RCBD (Randomized complete block design), use the least squires method to obtain the estimates $\hat{\mu}$, $\hat{t}_i$, and $\hat{b}_j$. **(6 Marks)**

e. Describe a 2-level $\frac{1}{8}$ fractional factorial experimental design and give circumstances under which it is applied. **(4 Marks)**

f. Briefly describe what is a crossover design and give an illustration **(5 Marks)**

g. Consider the layout below of a design.

B_1	$t_1(13)$	$t_2(17)$	$t_3(20)$
B_2	$t_3(10)$		
B_3	$t_1(37)$	$t_2(43)$	

Calculate the adjusted treatment totals for the treatments in the design. **(5 Marks)**

QUESTION TWO (20 MARKS)

a. Define a balanced incomplete block design (BIB) and give the conditions for its existence. **(4 Marks)**

b. Show that in a balanced incomplete block design (BIB) parameters v, b, r, k and λ satisfy the relations

$$\lambda = r\frac{(k-1)}{(v-1)}$$

(3 Marks)

c. A balanced incomplete block design experiment had yields as given below.

	BLOCKS					
TREATMENTS	1	2	3	4	5	6
1				18	19	15
2	13	14			11	
3	27		30			35
4		6	10	12		

i. Calculate the adjusted treatment totals. **(3 Marks)**

ii. Construct the ANOVA table for testing
$H_O: t_1 = t_2 = \ldots\ldots\ldots\ldots\ldots = t_v$ **(4 Marks)**

iii. Construct the ANOVA table for testing
$H_O: b_1 = b_2 = \ldots\ldots\ldots\ldots\ldots = b_b$ **(4 Marks)**

iv. Find the adjusted R^2 for the model and comment on it. **(2 Marks)**

QUESTION THREE (20MARKS)

An article in the *AT & T Technical Journal* (March/April 1986, Vol. 65, pp. 39-50) describes the application of two-level factorial designs to integrated circuit manufacturing. A basic processing step is to grow an epitaxial layer on polished silicon wafers. The wafers mounted on a susceptor are rotated and heat is applied until the epitaxial layer is thick enough. An experiment was run using two factors:

arsenic flow rate (A) and deposition time (B).Four replicates were run and the epitaxial layer thickness was measured (in μm). The data are shown below

		Replicate					Factor Levels	
A	*B*	I	II	III	IV		Low (−)	High (+)
−	−	14.037	16.165	13.972	13.907	*A*	55%	59%
+	−	13.880	13.860	14.032	13.914			
−	+	14.821	14.757	14.843	14.878	*B*	Short	Long
+	+	14.888	14.921	14.415	14.932		(10 min)	(15 min)

a.

i. Find the average effects $\hat{A}$, $\hat{B}$, $\widehat{AB}$ **(3 Marks)**

ii. Fit a regression model for the experiment. **(2 Marks)**

b. Take the **replicates I, II, III, and IV as BLOCKS**.

i. Find sum of squires (SS) due to factors A (SS_A), factors B (SS_B), SS_T, SS BLOCKS,SSE and interaction effect AB (SS_{AB}). **(9 Marks)**

ii. Construct the ANOVA table and test whether the data provide sufficient evidence to indicate the factors are statististically significant to the growth on an epitaxial layer on polished silicon wafers. **(4 Marks)**

iii. Find R^2 for the model and comment on the goodness of fit **(2 Marks)**

QUESTION FOUR (20 MARKS)

a. What is Confounding in factorial designs? **(3 Marks)**

b. Johnson and Leone (***statistics and Experimental Design in engineering and the Physical Sciences, Wiley, 1977***) describe an experiment to investigate warping of copper plates. The factors studied were the temperature and the copper content of the plates. The response variable was a measure of the amount of warping. The data were as follows:

Temperature (°C)	Copper Content (%)			
	40	60	80	100
50	17, 20	16, 21	24, 22	28, 27
75	12, 9	18, 13	17, 12	27, 31
100	16, 12	18, 21	25, 23	30, 23
125	21, 17	23, 21	23, 22	29, 31

i. Is there any indication that either factor affects the amount of warping? Is there any interaction between the factors? Use $\alpha = 0.05$ **(14 Marks)**

ii. Find adjusted R^2 for the model and comment on the goodness of fit **3 Marks)**

QUESTION FIVE (20 MARKS)

a. Differentiate between a general factorial experimental design and a symmetrical factorial design **(3 Marks)**

b. Consider a Partially Balanced Incomplete block design (PBIB) with v-treatments, b-blocks where each block size is k plots;

i. Briefly state the conditions for its existence. **(3 Marks)**

ii. Give an illustration for the design. **(3 Marks)**

c. Show that the expectation of treatment sum of squares of a completely randomized block design (CRBD) results to;

i. $(V-1)\,\sigma^2 + b\sum_{i=1}^{v} t_{i^2}$ **(6 Marks)**

ii. $(b-1)\,\sigma^2 + v\sum_{j=1}^{b} b_{j^2}$ **(1 Mark)**

iii. $(V-1)\,(b-1)\,\sigma^2$ **(4 Marks)**