LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST SEMESTER 2025/2026 ACADEMIC YEAR

FIRST YEAR EXAMINATION FOR THE DEGREES OF BACHELOR OF SCIENCE (STATISTICS) AND BACHELOR OF SCIENCE (COMPUTER SCIENCE)

MATH 121: CALCULUS I

STREAM: R ***TIME: 2 HRS***

DAY: WEDNESDAY [11.30 – 1.30 P.M] ***DATE: 04/02/2026***

THIS QUESTION PAPER CONSISTS OF FIVE (5) PAGES

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INSTRUCTION:

Answer all the questions in Section A and two questions from Section B.

SECTION A

QUESTION ONE (30 MARKS)

a) Let $f(x)=\begin{cases} \dfrac{x^2-9}{x-3}, x<3 \\ cx^2+10, x\geq 3 \end{cases}$

Find the value of c so that $f(x)$ is continuous at $x=3$. **(3 Marks)**

b) Find the roots of the following functions:

(i) $f(x)=x^2+3x+2$ **(2 Marks)**

(ii) $f(x)=x^3-2x^2-5x+6$ **(3 Marks)**

c) Evaluate the following limits:

(i) $\lim\limits_{x\to 2} 3x^2-4x+7$ **(2 Marks)**

(ii) $\lim\limits_{x\to 5} \dfrac{x^2-3x-10}{x-5}$ **(3 Marks)**

(iii) $\lim\limits_{x\to\infty} \dfrac{4y^3-3y^2+5y-9}{5y^3+2y^2-6y+3}$ **(3 Marks)**

d) Differentiate the following with respect to x:

(i) $y=xe^x$ **(2 Marks)**

(ii) $y=\tan x$ **(3 Marks)**

(iii) $y=(2x-5)^{10}$ **(3 Marks)**

(iv) Given that $x=t^3-t$ and $y=4-t^2$ **(2 Marks)**

e) Find the second derivative of the following:

(i) $g(w)=e^{1-2w^3}$ **(2 Marks)**

(ii) $f(t)=\ln(1+t^2)$ **(2 Marks)**

SECTION B

QUESTION TWO (20 MARKS)

a) Given that $f(x)=x^2$ and $g(x)=3x+1$. Find:

(i) $f \circ g$ **(2 Marks)**

(ii) The inverse of $g \circ f$ **(3 Marks)**

b) Find the value of a such that $\lim_{x\to -1}\frac{2x^2-ax-14}{x^2-2x-3}$ exists. What is the value of the limit?

(3 Marks)

c) Let $G(x)=\begin{cases}\frac{1}{(x+3)^2}, x\le -1\\ 2-x, -1<x\le 1\\ \frac{3}{x+2}, x>1\end{cases}$

Find all values of x where G is not continuous. **(4 Marks)**

d) Given that $f(x)=\begin{cases}\frac{\sqrt{9x^4+x^2}}{5x^2+3x+1}, x\le 0\\ x, x<0\end{cases}$

Is f continuous at $x=0$? **(4 Marks)**

e) The gross domestic product (GDP) of a certain country was $N(t)=t^2+5t+106$ billion dollars t years after 1990.

(i) At what rate was the GDP changing with respect to time in 1998? **(2 Marks)**

(ii) At what percentage rate was the GDP changing with respect to time in 1998?

(2 Marks)

QUESTION THREE (20 MARKS)

a) Find the limits of the following trigonometric functions

(i) $\lim_{x\to 0}\frac{\sin 3x}{\tan 5x}$ **(2 Marks)**

(ii) $\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}$ **(3 Marks)**

(iii) $\lim_{x\to 0}\frac{1-\cos 2x+\tan^2 x}{x\sin x}$ **(3 Marks)**

b) Use the power rule to determine $f'(x)=\sqrt[3]{2x-4}$. Then find an equation of the line tangent to the graph of f at the point $(6,2)$. **(4 Marks)**

c) Determine the stationary points of the curve $y=3x^4-8x^3+6x$. Hence

(i) Find the Coordinates of the stationary points. **(3 Marks)**

(ii) Classify the stationary points. **(3 Marks)**

(iii) Sketch the curve. **(2 Marks)**

QUESTION FOUR (20 MARKS)

a) Define a function. **(1 Mark)**

b) Use $y=\sqrt{x}$ to approximate the value of $\sqrt{1.1}$ given that $x=1$. **(4 Marks)**

c) A distance-time graph is represented by the equation $S=2t^3-2t^2-3t$.

Evaluate;

(i) Velocity at the time $t=2s$ **(2 Marks)**

(ii) Acceleration at time $t=1$ and $t=3$. **(2 Marks)**

(iii) Show that the minimum ever attained occurs when $t=\dfrac{2+\sqrt{22}}{6}$ **(3 Marks)**

d) Find the points of the curve $y=x^4+4x^3-3$ at which the gradient is zero, hence determine the nature of the points. **(8 Marks)**

QUESTION FIVE (20 MARKS)

a) A 2% error made in measuring the radius of a sphere. Find the percentage error in the volume. **(4 Marks)**

b) A ball is thrown vertically upwards. At time t seconds, its height h meters is given by $h=20t-5t^2$. Calculate the ball's maximum height above the ground. **(3 Marks)**

c) One side of a rectangle sheep pen is formed by a hedge. The other three sides are made using fencing. The length of the rectangle is x metres; 120 metres of fencing is available.

(i) Show that the area of the rectangle is $\frac{1}{2}x(120-x)m^2$. **(2 Marks)**

(ii) Calculate the maximum possible area of the sheep pen. **(2 Marks)**

d) Gas is escaping from a spherical balloon at the rate of $300cm^3/s$. How fast is the surface area shrinking when the radius is $300cm$? **(3 Marks)**

e) A particle travels in a straight line so that at time t seconds after the start, its distance $f(t)$ from a fixed point zero on the straight line is given by $f(t)=t^3-4t^2+12$. Evaluate:

(i) The velocity after 2 seconds. **(3 Marks)**

(ii) What is the acceleration after 4 seconds? **(3 Marks)**