LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST SEMESTER 2025/2026 ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE DEGREES OF BACHELOR OF SCIENCE (STATISTICS) AND BACHELOR OF SCIENCE (ECONOMICS AND STATISTICS)

STAT 312: THEORY OF ESTIMATION

STREAM: R ***TIME: 2 HRS***

DAY: FRIDAY [11.30 – 13.30 P.M] ***DATE: 30/01/2026***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS

- Answer question **ONE** and any other **TWO** questions.

QUESTION ONE (30 MARKS)

a) Define the following terms as used in Theory of Estimation

i. Unbiased estimator **(2 Marks)**

ii. Pivotal quantity **(2 Marks)**

b) Suppose that $\hat{\theta}_1$ is an estimator for θ such that $E\left(\hat{\theta}_1\right) = a\theta + b$, where $a \neq 0$. Show that $\hat{\theta}_2 = \dfrac{\hat{\theta}_1 - b}{a}$ is an unbiased estimator for θ. **(3 Marks)**

c) Let $X_1, X_2, ..., X_n$ be a random sample from a Geometric(θ) distribution, with pdf

$f(x;\theta) = \begin{cases} \theta(1-\theta)^x, x = 0,1,.. \\ 0, elsewhere \end{cases}$ where θ is unknown. Find the Maximum Likelihood Estimator of θ based on this sample. **(4 Marks)**

d) Let $X_1, X_2, ..., X_n$ be gamma random variables with parameter α and θ such that $E(X) = \alpha\theta$ and $Var(X) = \alpha\theta^2$. Find the Method of Moments Estimators for α and θ **(5 Marks)**

e) Let $X_1, X_2, ..., X_n$ be a random sample from the Poisson distribution with density function given as

$$f(x;\lambda) = \begin{cases} \dfrac{\lambda^x e^{-\lambda}}{x!}, & x = 0,1,2,... \\ 0, & elsewhere \end{cases}$$

and $\lambda > 0$ is unknown parameter.

i. Show that $\bar{X}$ is a consistent estimator of λ **(4 Marks)**

ii. Use Neyman factorization theorem to find a sufficient statistic for λ **(3 Marks)**

f) Verify whether a binomial distribution with parameter p belongs to the exponential class of distributions **(4 Marks)**

g) Let $X_1, X_2, ..., X_n$ be a random sample from the exponential distribution with pdf $f(x;\theta) = \begin{cases} \theta e^{-\theta x}, x > 0 \\ 0, elsewhere \end{cases}$ and unknown parameter θ. Find the estimator of θ by using the Method of Moments. **(3 Marks)**

QUESTION TWO (20 MARKS)

a) Define Sufficiency in relation to estimators **(2 Marks)**

b) State the Neyman factorization theorem used to obtain sufficient statistics **(3 Marks)**

c) Let $X_1, X_2, ..., X_n$ be a random sample for a population with probability mass function

$$f(x;\theta) = \begin{cases} \theta(1-\theta)^x, & x = 0,1,... \\ 0, & elsewhere \end{cases}$$

where $0 < \theta < 1$. Find the Sufficient statistic for θ **(3 Marks)**

d) Let $X_1, X_2, ..., X_n$ be a random sample form the normal distribution with unknown parameters mean μ and variance σ^2.

i. Find the Maximum Likelihood Estimators of μ and σ^2 **(4 Marks)**

ii. Find the Mean Squared Error of $\hat{\sigma}^2$ in part (i) above. **(8 Marks)**

QUESTION THREE (20 MARKS)

a) Let $X_1, X_2, ..., X_n$ be a random sample form the normal distribution with parameters mean μ and variance σ^2. Show that;

i. Both $\frac{1}{2}(X_1 + X_2)$ and X_4 are unbiased estimators of μ **(4 Marks)**

ii. $\frac{1}{2}(X_1 + X_2)$ is not a consistent estimator of μ **(5 Marks)**

b) Let $X_1, X_2, ..., X_n$ be a random sample from a population that is uniformly distributed with parameters a and b. It is known that $E(X) = \frac{a+b}{2}$ and $Var(X) = \frac{(b-a)^2}{12}$. Find the Method of Moments Estimator of a and b **(11 Marks)**

QUESTION FOUR (20 MARKS)

a) Explain the following

i. Mean Squared Error Consistency **(3 Marks)**

ii. Uniformly Minimum Variance Unbiased Estimator (UMVUE) **(4 Marks)**

b) Let $X_1, X_2, ..., X_n$ be a random sample from a population that is normally distributed with unknown mean μ and known variance σ^2.

i. Find the Cramer – Rao lower bound for the variance of unbiased estimators of μ

(8 Marks)

ii. Find the UMVUE for μ **(5 Marks)**

QUESTION FIVE (20 MARKS)

a) Let $X_1, X_2, ..., X_n$ be a random sample from $N(\mu, \sigma^2)$ where σ_1^2 and σ_2^2 are unknown but $\sigma_1^2 = \sigma_2^2 = \sigma$. Construct the $100(1-\alpha)\%$ confidence interval for $\mu_1 - \mu_2$. **(10 Marks)**

b) Let $X_1, X_2, X_3, ..., X_n$ be a random sample from a Poisson density given as;

$$f(x) = \frac{e^{-\theta}\theta^x}{x!},\ x = 0,1,2,... \quad \text{where } \theta > 0$$

Assume the prior distributionis given by,

$$g(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)}\ \theta^{\alpha-1}e^{-\theta\beta}\ , 0 < \theta < \infty, \text{where } \alpha \text{ and } \beta \text{ are known.}$$

i. Find the posterior distribution of θ. **(7 Marks)**

ii. Find the Bayes estimator of θ. **(3 Marks)**