LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

1ST SEMESTER 2022/2023 ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE DEGREES OF BACHELOR OF SCIENCE (STATISTICS), BACHELOR OF SCIENCE (GENERAL)/BACHELOR OF EDUCATION (SCIENCE)

MATH 312: REAL ANALYSIS I

STREAM: R ***TIME: 2 HRS***

DAY: WED [8.30AM – 10.30AM] ***DATE: 14/12/2022***

THIS QUESTION PAPER CONSISTS OF THREE (3) PAGES

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QUESTION ONE (30 Marks)

(a) Let $f(x)=\begin{cases} \sin\left(\frac{1}{x}\right) & , if\ \ x \neq 0 \\ 0, \ if\ \ x=0 \end{cases}$. And let $x_n = \frac{2}{(2n+1)\pi}$, show that $Lim_{n\to\infty} f(x_n) \neq f(Lim_{n\to\infty} x_n)$. (6 marks)

(b) Use $\varepsilon - \delta$ definition; define the derivative of a function f. Use an example; show that continuity does not always imply differentiability. (8 marks)

(c) Define the following terms as used in the real number system. (i)Lease upper Bound. (ii) Greatest lower bound. (6 marks)

(d) Define a Cauchy sequence. Then show that $\{a_n b_n\} \to ab \ \ for \{a_n\} \to a \ \ and \ \ \{b_n\} \to b$ (6 marks)

(e) Prove that $\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$, using induction principle. (4 marks)

QUESTION TWO (20 Marks)

(a) Let $f: Z^+ \to \Re$, where Z^+ is the set of positive integers, and f be a sequence of real numbers defined by. $\{x_n\} = \{1+\frac{1}{2n}\} \to 1$. Show that it converges. (4 marks)

(b) Show that $\{a_n + b_n\} \to a+b \ \ given\, that \ \ \{a_n\} \to a \ \ and \ \ \{b_n\} \to b$. (6 marks)

(c) Prove that every convergent sequence $\{x_n\}$ of $\Re$, is bounded. (5 marks)

(d) Show that $\{y_n\}$ converges to L if and only if every subsequence of $\{y_n\} \to L$. (5 marks)

QUESTION THREE (20 Marks)

(a) Define an absolute value of a, hence show that $|a+b| \leq |a|+|b| \quad \forall \ \ a,b \in$ (5 marks)

(b) Let E be a bounded subset of $\Re$, and $q = \sup E$. Show that q is unique. (5 marks)

(c) Find $D(f)$ and $R(f)$ given

(i) $f(x) = \frac{2}{x^2+1}$. (2 marks)

d) Suppose $f(x) = \frac{x^2-4}{x-2}$, Show that f is 1-1 but not onto

(ii) Suppose $f: A \to B$ *and* $g: B \to C$ if f and g are 1-1. Show that gof is a 1-1 function.

(8 marks)

QUESTION FOUR (20 Marks)

(a) Let $E \in \Re$ and $f: E \to \Re$ be a function, let $p \in E,$ then define continuity of f at p.

(4 marks)

(b) Let f be defined by $\left\{ \begin{array}{ll} \dfrac{x^2-4}{x-2} & ,x \neq 2 \\ 4 \quad , & x = 2 \end{array} \right\}$ Show that $x = 2$ is a removable discontinuity.

Also give an example of a 3rd kind discontinuity. (8 marks)

(c) Let $f(x) = x^2,$ and let $\{x_n\} = \{2 - \frac{1}{n}\}$. Show that $\lim_{n\to\infty} f(x_n) = f(\lim_{n\to\infty} x_n)$. (4 marks)

(d) Let $f(x) = \dfrac{2x^2-8}{x-2}$. Show that $\lim_{x\to 2} f(x) = 8$. (4 marks)

QUESTION FIVE (20 Marks)

(a) Let F be an ordered field. For $a, b \in F$, Show that (i)If $a < b,$ *then* $a + c < b + c$ (ii) If $a < b$ *and* $b < c,$ *then* $a < c$. (6 marks).

(b) Let f and g be functions defined on (a,b) that are differentiable at $c \in (a,b)$. Then show that $(\alpha f + \beta g)'(c) = \alpha f'(c) + \beta g'(c)$ for any real numbers α and β.

(8 marks)

(c) Let f and g be functions defined on (a,b) that are differentiable at $c \in (a,b)$ *show that* $(\alpha f + \beta g)^1(c) = \alpha f^1(c) + \beta g^1(c)$,where α, β are real numbers.

(6marks)