LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST SEMESTER 2025/2026 ACADEMIC YEAR

FIRST YEAR EXAMINATION FOR THE DEGREE OF

BACHELOR OF EDUCATION (SCIENCE/ARTS)

STAT 121: INTRODUCTION TO PROBABILITY AND STATISTICS II

STREAM:* *BEd (SCIENCE/ARTS) ***TIME: 2 HRS***

DAY: FRIDAY [8.30 – 10.30 A.M] ***DATE: 06/02/2026***

THIS QUESTION PAPER CONSISTS OF FIVE (5) PAGES

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INSTRUCTIONS:

Answer Question **ONE** and any other **TWO** questions.

Mobile phones are not allowed in the examination room.

Candidates are not permitted to write on the examination question paper.

QUESTION ONE (30 MARKS)

(a) Let the random variable X have the probability density function

$$f_X(X) = 6x - 6x^2, \; 0 < x < 1.$$

(i) Find an explicit expression for $F_X(x)$, the CDF of the random variable X, and then use this result to find the numerical value of $P(0.6 < X < 0.8)$. **(5 Marks)**

(ii) Calculate the value of $Var(2X + 3)$. **(4 Marks)**

(b) Claim sizes in a certain insurance situation follows a distribution with moment generating function (M.G.F.) given by $M(t) = (1 - 10t)^{-2}$. Find the value of $E(X^2)$ and use it to evaluate $E(3X^3 - 5)$. **(4 Marks)**

(c) If $X \sim B(20, 0.4)$, find approximations to $P(6 \leq X \leq 8)$ using the

(i) Normal distribution **(3 Marks)**

(ii) Poisson distribution **(2 Marks)**

(d) The random variable X has probability density function

$$f(x) = k(1 - x)(1 + x), \qquad 0 < x < 1,$$

where k is a positive constant.

(i) Find the value of the constant k. **(2 Marks)**

(ii) Calculate the probability $P(X > 0.25)$. **(2 Marks)**

(e) Suppose that the random variable X is distributed as $P(\lambda)$; that is,

(i) Suppose that $f(2) = 2f(0)$. Determine the value of the parameter λ. **(2 Marks)**

(ii) Suppose now that $P(X = 0) = 0.1$. Calculate the probability $P(X = 5)$. **(2 Marks)**

(f) If the distribution function of X is given by

$$F_X(x) = \begin{cases} 0, & x < 0, \\ 1/2, & 0 \le x < 1, \\ 3/4, & 1 \le x < 2, \\ 4/5, & 2 \le x < 3, \\ 9/10, & 3 \le x < 3.5, \\ 1, & 3.5 \le x. \end{cases}$$

Calculate the probability mass function of X. **(4 Marks)**

<u>QUESTION TWO (20 MARKS)</u>

(a) The random variable X has the probability density function

$$f_X(x) = \begin{cases} k(1/x)^{k+1}, & 1 \le x < \infty, \quad k > 2 \\ 0, & elsewhere \end{cases}$$

(i) Prove that $f_X(x)$ is a valid probability distribution function. **(3 Marks)**

(ii) Find the cumulative distribution function (CDF) of X denoted by $F_X(x)$. **(3 Marks)**

(iii) What is the variance of X. **(5 Marks)**

(b) A discrete random variable X has a cumulative distribution function (CDF) with the following values:

Observation	10	20	30	40	50
CDF	0.5	0.7	0.85	0.95	1

Calculate the probability that X takes a value:

(i) Larger that 10. **(1 Mark)**

(ii) Exactly 40. **(1 Mark)**

(iii) Larger than 20 but less than 50. **(2 Marks)**

(iv) Exactly 20 or exactly 40 using both the CDF and the probability mass function. Compare the two answers. **(5 Marks)**

QUESTION THREE (20 MARKS)

(a) Suppose that the continuous random variable X has the distribution

$$f_X(x) = ke^{-(x-\theta)^{2m}}, -\infty < \theta < \infty, \qquad k > 0,$$

where m is a known positive integer.

(i) For r a non-negative integer, develop an explicit expression for $E[(X-\theta)^{2r}]$. (Hint: Let $u = (x-\theta)^{2m}$ so that $(x-\theta) = u^{(1/2m)}$ and $dx = \frac{1}{2m}u^{(1/2m)-1}du$.) **(7 Marks)**

(ii) When $m = 1$ and $r = 1$, determine the numerical value of $E[(X-\theta)^{2r}]$, and provide a rationale for why this numerical value makes sense. **(3 Marks)**

(b) Let the random variable X have the Geometric p.d.f.

$$f(X) = pq^{x-1}, \ x = 1,2,3,\dots \quad (q = 1-p).$$

(i) What is the probability that the first success will occur by the 10^{th} trial if $p = 0.2$? **(3 Marks)**

(ii) Find the variance of the distribution. **(7 Marks)**

QUESTION FOUR (20 MARKS)

(a) In an undergraduate statistics class of 80, 10 of the students are actually graduate students. If 5 students are chosen at random from the class, what is the probability that:

(i) No graduate students are included? **(3 Marks)**

(ii) At least 3 undergraduate students are included? **(3 Marks)**

(b) Consider the discrete random variable X with probability function

$$f(x) = \frac{4}{5^{(x+1)}}, \quad x = 0,1,2,\dots$$

(i) Show that the moment generating function of the distribution of X is given by

$$M_X(t) = \frac{4}{(5-e^t)}, \quad e^t < 5.$$ **(5 Marks)**

(ii) Determine $Var(X)$ using the moment generating function given in part (i). **(5 Marks)**

(c) Imperfections in a computer circuit boards and computer chips lend themselves to statistical treatment. For a particular type of board, the probability of a diode failure is 0.05 and the board contains 100 diodes. The board will not work if there are defective diodes. What is the probability that a board will not work? **(4 Marks)**

QUESTION FIVE (20 MARKS)

(a) Suppose that the duration in minutes of long-distance telephone conversation follows an exponential density function;

$$f(x) = \frac{1}{2}\, e^{-x/5} \text{ for } x > 0.$$

Find the probability that the duration of a conversation:

(i) Will exceed 5 minutes. **(3 Marks)**

(ii) Will be between 3 and 6 minutes. **(3 Marks)**

(iii) Will be less than 6 minutes given that it was greater than 3 minutes. **(4 Marks)**

(b) Suppose that the real-valued random variable X is normally distributed with parameters $\mu = 3$ and $\sigma^2 = 4$. Calculate the

(i) probability $P(-2 \leq X \leq 3)$. **(3 Marks)**

(ii) value of a such that $P(X - 2 < a) = 0.95$. **(3 Marks)**

(c) The daily amount of coffee, in litres dispensed by a machine located in an airport lobby is a random variable X having a continuous uniform distribution

$$f(X) = \begin{cases} 1/(b-a), & a \leq X \leq b \\ 0 & elsewhere \end{cases}$$

With $a = 7$ and $b = 10$. Find the probability that on a given day the amount of coffee dispensed by this machine will be

(i) at most 8.8 litres. **(2 Marks)**

(ii) more than 7.4 litres but less than 9.5 litres. **(2 Marks)**