LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

2ND SEMESTER 2023/2024 ACADEMIC YEAR

FIRST YEAR EXAMINATION FOR THE DEGREE OF SCIENCE IN ECONOMICS AND STATISTICS

ECON 123: MATHEMATICS FOR ECONOMISTS II

STREAM: ***ECON/STAT*** ***TIME: 2 HRS***

DAY: WEDNESDAY [8.30-10.30 A.M] ***DATE: 10/04/2024***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS:

1. Answer Question ONE (compulsory) and ANY OTHER TWO questions.
2. Question one carries 30 marks and the rest carry 20 marks each.
3. Clearly show your working.

QUESTION ONE (30 MARKS)

a) Differentiate $y = (x^2 - 4x + 5)^3$ **(3 marks)**

b) Given $M = \begin{bmatrix} 1 & 2 & 5 \\ 6 & 4 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, prove that $detM = detM^T$ **(4 marks)**

c) i) Set up the Lagrangian function for a firm that can buy two inputs labour (L) and capital (K) at KSh w per unit and KSh r per unit respectively and faces the production function $Q = L^{\frac{2}{3}}K^{\frac{1}{3}}$ with a budget of KSh C. **(3 marks)**

ii) Maximize the output in (i) subject to the given factor cost constraint. **(4 marks)**

d) i) Find the Hessian matrix for the function

$$f(x, y) = 6x - x^2 + 16y - 4y^2$$ **(4 marks)**

ii) Interpret the Hessian matrix in (i). **(1 mark)**

iii) The optimum value of f(x, y). **(2 marks)**

e) For the function

$$z = x^2 - 9 - 2y^2$$

i) Find the critical values of x and y. **(4 marks)**

ii) Determine whether the critical point is a maximum or minimum. **(3 marks)**

Iii) Find the value of the extreme point. **(2 marks)**

QUESTION TWO (20 MARKS)

An individual has the utility function $U = {q_1}^2{q_2}^2$, a budget of Kshs. 40, and can buy good q_1 at Kshs. 2 per unit and good q_2 at Kshs. 4 per unit.

a) Find the combination of q_1 and q_2 that he/she should purchase to maximise utility. **(6 marks)**

b) Using the bordered Hessian matrix, prove that the second-order conditions for utility maximization are met. **(3 marks)**

c) What is the maximum utility? **(3 marks)**

d) Given a two commodity market model:

$Qd_1 = 8 - 2P_1 + P_2$ $\qquad Qd_2 = 16 + P_1 - P_2$

$Qs_1 = -5 + 3P_1$ $\qquad Qs_2 = -1 + 2P_2$

Use the inverse matrix method to find the value of equilibrium:

i. Prices. **(5 marks)**

ii. Quantities. **(3 marks)**

QUESTION THREE (20 MARKS)

a) Use Lagrangian method to Maximize the utility function $U(X,Y) = lnX + lnY$ subject to the budget constraint $P_X + P_Y = I$. **(5 marks)**

b) Construct the bordered Hessian matrix for the Lagrangian function in (a). **(4 marks)**

c) Find the determinant of the bordered Hessian matrix in (b). **(2 marks)**

d) Hence establish whether the first order condition values for the choice variables give maximum or minimum point. **(1 mark)**

e) The following is a system of IS and LM equations:

$$IS: f(X_1, X_2, \alpha) = 0$$
$$LM: g(X_1, X_2, \alpha) = 0$$

i) Find the total derivatives for the IS and LM functions. **(2 marks)**

ii) Determine the Jacobian matrix. **(3 marks)**

iii) Hence solve the system of total derivatives using Cramer's rule. **(3 marks)**

QUESTION FOUR (20 MARKS)

a) Differentiate,

i. $y = x^3(3 - 4x^2)$ **(4 marks)**

ii. $y = \frac{x^2-1}{x-1}$ **(4 marks)**

b) Consider the following input-output table (figures in $ millions) for a hypothetical economy;

	Agriculture	Industry	Services	Final Demand	Gross Production
Agriculture	10	60	35	55	160
Industry	65	240	55	90	450
Services	25	70	45	85	225

Imports (M)	15	30	55		
Value added (V)	45	50	35		
Total Value of Inputs	160	450	225		

i) Determine GNP by factor payments approach. **(3 marks)**

ii) Does your result agree with the value of GNP calculated by expenditure approach? **(4 marks)**

iii) If exports constitute 20 per cent of final demand, what is the balance of trade for this country? **(5 marks)**

QUESTION FIVE (20 MARKS)

a) Use the Gauss-Jordan method to solve following system of simultaneous equations derived from a three commodity market:

$$x_1 + 6x_2 - x_3 = 10$$
$$2x_1 + 3x_2 + 3x_3 = 17$$
$$3x_1 - 3x_2 - 2x_3 = -9$$

(10 marks)

b) Maximize $XY(9 - X - Y)$
Subject to $X + Y \leq 5$

i) Set up the Lagrangian function. **(1 mark)**

ii) Set up the Khun –Tucker conditions which must be satisfied by the solution of the Lagrangian function. **(4 marks)**

iii) Determine the values of x and y. **(5 marks)**