



UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

**FOURTH YEAR EXAMINATION FOR THE DEGREE OF
BACHELOR OF SCIENCE (GENERAL)**

MATH 428: NUMERICAL ANALYSIS II

STREAM: R

TIME: 2 HRS

DAY: THURSDAY [2.30 – 4.30 P.M]

DATE: 18/04/2024

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS: Answer Question **ONE** and Any Other **TWO** Questions

QUESTION ONE (30 MARKS)

- (a). Solve the following system by the method of Triangulation (LU factorization)

$$x + 5y + z = 14$$

$$2x + y = 3z = 13$$

$$3x + y + 4z = 17$$

(5 Marks)

- (b). Use Taylor method to compute $y(0.2)$ correct to 4 decimal places given $\frac{dy}{dx} = 1 - 2xy$
and $y(0) = 0$

(5 Marks)

- (c). Apply Gauss two point formula to evaluate $\int_{-1}^1 \frac{dx}{1+x^2}$

(5 Marks)

- (d). Find the dominant Eigen value of $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ by power method and hence find the other Eigen value. Verify your results using any other matrix theory

(5 Marks)

- (e). Find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ by the Gauss Jacobi method.

(5 Marks)

- (f). Find the least squares line approximating the data given in the table below

x_i	1	2	3	4	5	6	7	8	9	10
y_i	1.3	3.5	4.2	5.0	7.0	8.8	10.1	12.5	13.0	15.6

(5 Marks)

QUESTION TWO (20 MARKS)

- (a). From the following table of values of $f(x)$ and $f'(x)$ obtain $P_n(x)$ using Hermite interpolation hence evaluate $f(0.5)$ **(10 Marks)**

x	-1	0	1
$f(x)$	1	1	3
$f'(x)$	-5	1	7

- (b). Use Gauss three point Quadrature to evaluate $\int_{0.2}^{1.5} e^{-x^2} dx$ **(5 Marks)**
- (c). Obtain the linear polynomials using the least square approximation with the weight function $w(x)$, for the continuous data defined by the function $f(x) = x^3$ on $(0,1)$ **(5 Marks)**

QUESTION THREE (20 MARKS)

$$45x_1 + 2x_2 + 3x_3 = 58$$

- (a). Find the solution of the system of Equations $-3x_1 + 22x_2 + 2x_3 = 47$ correct to three decimal places, using Gauss-Seidel iteration method (Perform four iterations)

(10 Marks)

- (b). Using Jacobi Method find the Eigen values and Eigen vectors of the matrix

$$\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}$$

(10 Marks)**QUESTION FOUR (20 MARKS)**

- (a). Derive the least square quadratic fit for the discrete data $(x_i, f_i), i = 0(1)n$ **(6 Marks)**

- (b). Fit a parabola by the method of least square to the following data, also estimate y at

$$x = 6$$

(8 Marks)

x	1	2	3	4	5
y	5	12	26	60	97

$$3x + 4y + 5z = 18$$

- (c). Use Gauss elimination to solve $2x - y + 8z = 13$

(6 Marks)

$$5x - 2y + 7z = 20$$

QUESTION FIVE (20 MARKS)

- (a). Find an approximation to $y(1.6)$ for the initial value problem $y' = x + y^2, y(1) = 1$

Using the Euler method with $h = 0.2$

(8 Marks)

- (b). Find all α so that $A = \begin{pmatrix} 2 & \alpha & -1 \\ \alpha & 2 & -1 \\ -1 & 1 & 4 \end{pmatrix}$ is positive definite

(6 Marks)

- (c). Find the condition number of the system and comment on the sensitivity of the system

$$\begin{pmatrix} 0.1 & 1.8 \\ 6.2 & 5.3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2.1 \\ 6.2 \end{pmatrix}$$

(6 Marks)

