



UNIVERSITY EXAMINATIONS

FIRST SEMESTER 2025/2026 ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE DEGREES OF
BACHELOR OF SCIENCE (GENERAL), BSC (ECON &
STATS) BACHELOR OF EDUCATION (SCIENCE/ARTS)

MATH 314: ORDINARY DIFFERENTIAL EQUATIONS I

STREAM: R

TIME: 2 HRS

DAY: TUESDAY [8.30 – 10.30 A.M]

DATE: 03/02/2026

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

(a) Complete the following table; (4 Marks)

Differential equation	Classification e.g ODE or PDE	Type e.g Linear or Non-Linear	Order	Degree
$\left(\frac{dy}{dx}\right)^4 - 3x\left(\frac{d^3y}{dx^3}\right)^5 = 0$				
$\frac{\partial^2 I}{\partial \beta^2} + \frac{\partial I}{\partial \alpha} = \sin \alpha \cos \beta$				

(b) Solve the following ordinary differential equations;

(i) $(x^2 + 2xy^3)dx + (y^3 + 3x^2y^2)dy = 0$ (3 Marks)

(ii) $(x - y)dx + xdy = 0, y(3) = 0$ (3 Marks)

(iii) $e^{-x} \cos y dx - \sin y dy = 0$ (3 Marks)

(c) Determine the differential equation associated with;

(i) $x^2 + 4xy^2 - y^3 = A$ where A is a constant (3 Marks)

(ii) $y = Ke^{3x} + M \cos(3x)$ Where K and M are constants (3 Marks)

(d) Solve the linear equation;

$$xy' + y = \sqrt{x}, y(1) = 2\frac{2}{3}$$
 (3 Marks)

(e) Use the method of undetermined coefficients to solve;

$$y'' - 2y' - 3y = 3e^{2t}$$
 (4 Marks)

(f) The population of Nyahururu town was 10,000 in the year 2015 and 30,000 in the year 2020. Using the exponential model for population growth, find the population of Nyahururu town in 2030. (Give your answer to the nearest person). (4 Marks)

QUESTION TWO (20 MARKS)

(a) Verify that $y = Ae^{3x} + Be^{-2x}$ is a solution of $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 6y$ (4 Marks)

(b) (i) Distinguish between the general solution and a particular solution giving one example of each. (4 Marks)

(ii) Solve $\frac{dx}{y+1} = (x-2)dy, y(3) = 2$ (4 Marks)

- (c) Show that that $(3x + 4y^2)dx + 4xydy = 0$ is not exact but has an integrating factor of the form x^n , where n is an integer. Hence or otherwise, solve it. (4 Marks)
- (d) A cup of coffee at 82°C is placed in a room with an ambient temperature of 21°C . After 2 minutes, the coffee has cooled to 74°C . How long will it take for the coffee to reach a drinkable temperature of 43°C ? (4 Marks)

QUESTION THREE (20 MARKS)

- (a) Solve the following differential equation, showing all work. Verify the solution you obtain.

$$y'' - 2y' + y = 0; \quad y(0) = 1, \quad y'(0) = -2. \quad (4 \text{ Marks})$$

- (b) Solve $y(1 + \sin^2 x)y' = (y^2 - 1)\sin x \cos x, y(0) = 7$ (4 Marks)

- (c) Given the following differential equation;

$$xy' + y = \frac{x}{1 + x^2}$$

Find its solution given that $y(1) = 3\ln 2$ (4 Marks)

- (d) Solve the equation $y' + \frac{2y}{x} = x^2 y^3$ (4 Marks)

- (e) Use the method of variation of parameters to solve $5y'' - 3y' - 2y = e^{3x}$ (4 Marks)

QUESTION FOUR (20 MARKS)

- (a) A quantity P increases at a rate proportional to itself. If P_0 is the initial quantity of P ;

(i) Show that $P = P_0 e^{kt}$, where k is a constant. (3 marks)

(ii) If $P_0 = 500$ and $P(9) = 4000$, find $P(6)$ (3 marks)

- (b) Find the general solution of the second order ordinary differential equations;

(i) $\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} - 35y = 0$ (3 marks)

(ii) $\frac{d^2 v}{dt^2} + 16v = 8\frac{dv}{dt}$ (3 marks)

(iii) $8\frac{dz}{d\theta} = \frac{d^2 z}{d\theta^2} + 25z$ (3 marks)

- (c) Determine whether $xydy = (y^2 - x^2)dx$ is homogenous or not? Hence or otherwise, solve it subject to the initial condition $y(1) = -2$. (5 marks)

QUESTION FIVE (20 MKS)

(a) Find the differential equation associated with $y = e^{2x}[A\cos(5x) + B\sin(5x)]$ where A and B are arbitrary constants. (4 marks)

(b) Decide whether $xsinydx - (x^2 - 5)cosydy=0$ is separable or not hence or otherwise solve it. (4 marks)

(c) Solve the Bernoulli equation $x \frac{dy}{dx} + y = y^2 x^2 \ln x$ (4 marks)

(d) The Newton's second law model for a vibrating spring with damping and no forcing is given by the equation,

$$my'' + by' + ky = 0,$$

Given that $m = 10$ kg, $b = 60$ kg/sec, $k = 250$ kg/sec², $y(0) = 0.3$,
and $y'(0) = -0.1$ m/sec.

(i) Find the equation of motion (3 marks)

(ii) What is the position of the mass after 1 second? (1 mark)

(e) After 24 days, 2.0 mg of an original 128.0 mg radioactive sample remain. What is the half-life of the sample? (4 marks)