



UNIVERSITY EXAMINATIONS

1ST SEMESTER 2022/2023 ACADEMIC YEAR

**FOURTH YEAR EXAMINATION FOR THE DEGREES
OF BACHELOR OF SCIENCE
(STATISTICS)/BACHELOR OF SCIENCE (ECONOMICS
& STATISTICS)**

STAT 411: APPLIED TIME SERIES ANALYSIS

STREAM: R

TIME: 2 HRS

DAY: FRIDAY [11.30AM – 13.30]

DATE: 09/12/2022

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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QUESTION ONE

(a) Consider the series $(1 - 0.8B)X_t = (1 - 0.5B)e_t$. Express the series as a moving average
(6 marks)

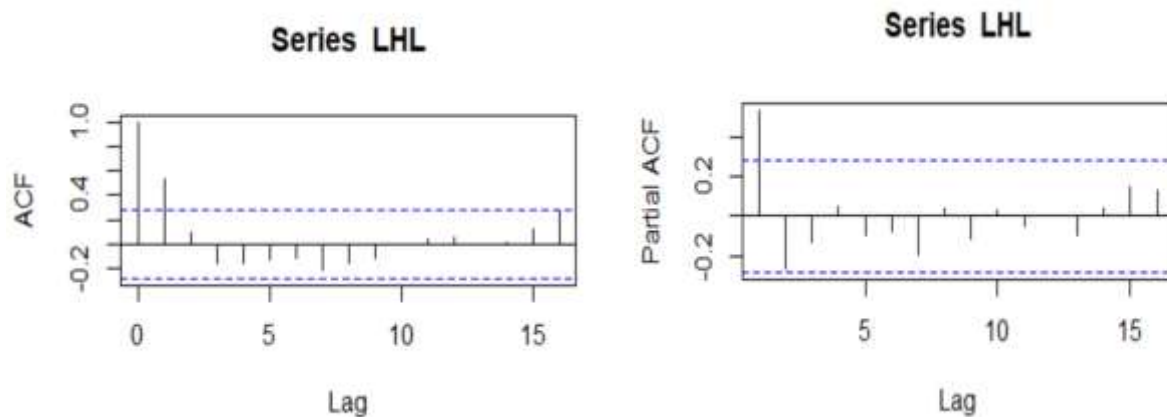
(b) Give examples of time series data in the following fields

- (i) Business
- (ii) Economics
- (iii) Official Statistics
- (iv) Natural sciences
- (v) Environment

(5 marks)

(c) Let $X_t = e_t + \phi_1 e_{t-1} + \phi_2 e_{t-2}$ where $e_t \sim iid N(0, \sigma^2)$. Find Autocovariance at lag 1, 2.
(6 marks)

(d) The ACF and the PACF are given below



(i) Determine the order of the tentative ARMA (p,d,q) model. (4 marks)

(ii) Use the AIC values for selected models are given below. Identify the most parsimonious model form the list. (4 marks)

AIC values for selected models

		MA(q)		
		0	1	2
AR(p)	0	78.25	64.84	61.98
	1	63.85	64.13	63.98
	2	62.77	62.86	65

(e) Identify appropriate ARIMA models from the sample ACF below. You can also calculate other identification measures. Justify your answer using theoretical ACF for ARIMA model.

(i) $N=100$

K	1	2	3	4	5	6	7	8
Pk	-0.35	-0.17	-0.09	-0.01	-0.01	-0.04	-0.07	0.09

(3 marks)

(ii) $n=100$

$$data = Z_t \quad W_t = (1-B)Z_t \quad \bar{w} = 35 \quad S_w^2 = 1500$$

K	1	2	3	4	5	6	7	8
$\hat{\rho}_z(k)$	0.99	0.98	0.98	0.97	0.94	0.91	0.89	0.88
$\hat{\rho}_w(k)$	0.45	-0.04	0.12	0.06	-0.18	0.16	0.07	0.05

(5 marks)

QUESTION TWO

(a) Consider each of the following models: -

(i) $(1-B)X_t = (1-1.5B)e_t$

(ii) $(1-0.8B)X_t = (1-0.5B)e_t$

(iii) $(1-0.6B)X_t = (1-1.2B+0.2B^2)e_t$

(iv) Verify whether the models are stationary, or invertible or both. (4 marks)

(v) Express the model in an MA representation if it exists (4 marks)

(b) Consider the time series model $(1-0.6B)X_t = (1-0.9B)e_t$. Find

i) the ACF ρ_k (6 marks)

ii) The PACF Ω_{kk} (6 marks)

QUESTION THREE (20 marks)

(a) Verify the following properties for the acf of a stationary process (4 marks)

(b) For the following model: $X_t = (1-0.7B+0.5B^2)e_t$

(i) Check if the model is stationary and invertible. (4 marks)

(ii) Find the ACF ρ_k , for $k = 1, 2, 3$ (6 marks)

(iii) Find the PAC, Q_{kk} , for $k = 1, 2, 3$ (6 marks)

QUESTION FOUR (20 marks)

For the AR (2) process, find the

- (i) The differential equation for auto correlationa function (2 marks)
 (ii)The partial autocorrelation at lag 1,2,3 (6 marks)

QUESTION FIVE (20 marks)

Assume that 100 observations from AR(2) model $X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + e_t$ gave the following sample ACF. $\hat{\rho}_1 = 0.8$ $\rho_2 = 0.5$ $\hat{\rho}_3 = 0.4$ $\gamma(0) = 132.4$ $\sigma^2 = 140$

(10 marks)

Estimate ϕ_1 and θ_1

(10 marks)

