LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

1ST SEMESTER 2022/2023 ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE DEGREES OF BACHELOR OF SCIENCE (STATISTICS)/BACHELOR OF SCIENCE (ECONOMICS & STATISTICS)

STAT 311: MATHEMATICAL STATISTICS III

STREAM: R ***TIME: 2 HRS***

DAY: THURS [8.30AM – 10.30AM] ***DATE: 08/12/2022***

THIS QUESTION PAPER CONSISTS OF FIVE (5) PAGES

PLEASE DO NOT OPEN UNTIL THE INVIGILATOR SAYS SO.

QUESTION ONE (30 marks)

a) Customers at a motorway service station enter the cafeteria through a turnstile. The cafeteria is open 24 hours a day and an automatic counting device records the number of people entering each minute. State, giving reasons, whether or not it is likely that these data will follow a Poisson distribution.

(2marks)

b) In a certain town, 40% of the eligible voters prefer candidate A, 10% prefer candidate B, and the remaining 50% have no preference. You randomly sample 10 eligible voters. What is the probability that 4 will prefer candidate A, 1 will prefer candidate B, and the remaining 5 will have no preference?

(4marks)

c) Suppose that **y**1, **y**2, . . . , **y**n is a random sample from a non-normal multivariate population with mean $\underline{u}$ and covariance matrix Σ. If n is large, what is the approximate distribution of each of the following?

i) $\sqrt{n}\,(\underline{y} - \underline{u})$ (2marks)

ii) $\bar{Y}$ (2marks)

d) Let x ~ $N_4(\mu,\Sigma)$, where

$$\mu = \begin{pmatrix} 5 \\ 6 \\ 7 \\ 8 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 3 & 2 & 0 \\ 1 & 2 & 4 & 0 \\ 0 & 0 & 0 & 9 \end{pmatrix}$$

Find the distribution of: -

(i) $\begin{pmatrix} x_2 \\ x_4 \end{pmatrix}$ (2marks)

(ii) $x_1 - x_4$ (4marks)

iv) Given that $\begin{matrix} z_1 = x_1 + x_2 \\ z_2 = x_1 - x_2 \end{matrix}$, Find the joint distribution of z_1 and z_2 (6marks)

d) It is given that $G_x(s) = \dfrac{ks}{1 - 0.2s^2}$ where k is a positive constant.

i) Determine the value of k (2marks)

ii) Obtain the mean and variance of x (6marks)

QUESTION TWO (20 marks)

a) Suppose that Y and X are two subvectors, such that $Y \ is \ 2\times1$ and $Y \ is \ 3\times1$, with $\underline{\mu} \ and \ \Sigma$ portioned accordingly

$$\underline{\mu} = \begin{pmatrix} 2 \\ -3 \\ 4 \\ -3 \\ 5 \end{pmatrix} \text{ and } \Sigma = \begin{pmatrix} 14 & -8 & 15 & 0 & 3 \\ -8 & 18 & 8 & 6 & -2 \\ 15 & 8 & 50 & 8 & 5 \\ 0 & 6 & 8 & 4 & 0 \\ 3 & -2 & 5 & 0 & 1 \end{pmatrix}$$

Assume that $\begin{pmatrix} y \\ x \end{pmatrix} \sim N_5(\underline{\mu}, \Sigma)$

b) Find E(y/x) when x= $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ (6marks)

c) Find cov(y/x) (8marks)

d) A factory puts biscuits into boxes of 100. The probability that a biscuit is broken is 0.03. Find the probability that a box contains 2 broken biscuits

(4marks)

QUESTION THREE 20 Marks

a) If $y \sim N(\mu_y, \Sigma_y)$, obtain the distribution of

i) Z=Ay, where A is a scalar (2marks)

ii) Var Z (4marks)

b) Derive the bivariate the normal distribution. (14marks)

QUESTION THREE (20 Marks)

a) A bowl has 2 maize marbles, 3 blue marbles and 5 white marbles. A marble is randomly selected and then placed back in the bowl. You do this 5 times. What is the probability of choosing 1 maize marble, 1 blue marble and 3 white marbles?

(4marks)

b) Explain two issues addressed by convergence theorems (4marks)

c) Distinguish between convergence in probability and convergence mean square (6marks)

d) Let X1, X2, . . . , Xn, . . . be i.i.d. with finite mean E(X) and variance Var(X). Let $S_n = \frac{\sum_{i=1}^{n} X_i}{n}$.

Show that Sn → E(X) in m.s (6marks)

QUESTION FOUR (20 Marks)

Table below contains data from O'Sullivan and Mahan (1966) with measurements of blood glucose levels on three occasions for 6 women. The *Y*'s represent fasting glucose measurements on the three occasions; the *X*'s are glucose measurements 1 hour after sugar intake.

	Fasting		One Hour after Sugar intake	
	Y1	Y2	X1	X2
	62	75	116	130
	74	64	109	101
	64	73	77	102
	73	73	115	110
	68	63	76	84
	69	82	72	133
	64	62	133	134
	70	76	150	158
Total	544	568	848	952
Mean	68	71	106	119

a) Find

i) The portioned mean vector

(4marks)

ii) Covariance matrix for all four variables partitioned $S = \begin{bmatrix} S_{yy} & S_{xy} \\ S_{yx} & S_{xx} \end{bmatrix}$

(8marks)

b) Let the n × 1 vector y ~ N (0, I). Find the distribution of $y'y$

(4marks)

c) What is the normal distribution approximation for the binomial distribution where $n = 20$ and $p = .25$? (4marks)

QUESTION FIVE (20 Marks)

a) State two applications of probability generating functions (2marks)

b) Derive the probability generating function of binomial distribution. (4marks)
Hence obtain the: -
i) mean (3marks)
ii) variance (4marks)

c) Sample problem Recent university graduates revealed that
- Probability of job related to field of study = 0.60
- Probability of job unrelated to field of study = 0.30
- Probability of no job = 0.10

Of 30 randomly chosen students,

i) what is probability that 15 are employed in a job related to their field of study, 10 are employed in a job unrelated to their field of study, and 5 are unemployed? (4marks)

ii) The covariance of job related study and no job. (3marks)