LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/20234ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (GENERAL)

MATH 311: ALGEBRA I

STREAM: R ***TIME: 2 HRS***

DAY: FRIDAY [11.30A.M -1.30 P.M] ***DATE: 12/04/2024***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS

Answer question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) An operation $*$ is defined by $x*y= x^{|y|}$, for all $x, y \in \mathbb{Z}$

i) Obtain $(x*y) *z$ when $x=2$, $y=-1$and $z=-3$ **(4 Marks)**

ii) Determine whether $*$ is associative in $\mathbb{Z}$ or not **(4 Marks)**

b) i) Evaluate all the right cosets of H={0, 6, 12}in G=($\mathbb{Z}_{18}$, + *mod* 18) **(3 Marks)**

ii) Calculate the value of x for which

$x+4=4^{-1}$ in $\mathbb{Z}_{18}$ **(3 Marks)**

c) i) Evaluate the equivalence classes determined by the relation R, on the set of integers, where $x \text{ R } y \Leftrightarrow y-x$ is a multiple of 5. **(4 Marks)**

ii) Solve for x in the equation

$2x^2+x-1=0 \ mod\ 5$ **(4 Marks)**

d) i) Express permutation q as a product of disjoint cycles

$q=(45)(135)(425)(1234)$ **(4 Marks)**

ii) Determine the orders of all possible subgroups of G if |G|=28 **(4 Marks)**

QUESTION TWO (20 MARKS)

Given the group $G = \mathbb{Z}_{17}$-{0}, under multiplication mod 17, and $H = \{1, 4, 13, 16\}$ a subset of G

i) Show that H is a subgroup of G **(5 Marks)**

ii) Evaluate x in H if
$13x^2=4$ **(5 Marks)**

iii) Compute the factor group G/H **(5 Marks)**

iv) Show that $G/H \cong \mathbb{Z}_4$ **(5 Marks)**

QUESTION THREE (20 MARKS)

a) The set S contains all vectors in $\Re^2$ whose first component is zero, S={$\boldsymbol{x}$: $\boldsymbol{x}$=(0, a), a $\epsilon\Re$}

Show that S is a group under vector addition **(5 Marks)**

b) Evaluate the group G, generated by the matrix
$A=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, under matrix multiplication **(5 Marks)**

c) The group $G=\{1, p, q, r\}$, under the binary operation $*$ is such that $pr=q^2=1$ and $r^2=p^2=q$.

i) Evaluate p^3

(5 Marks)

ii) Determine the order of each element in G and use your results to investigate whether G is cyclic **(5 Marks)**

QUESTION FOUR (20 MARKS)

a) Given the groups G_1={ all 2×2 invertible matrices with real entries}, under matrix multiplication and $G_2 = \Re$-{0}, under multiplication of real numbers, the mapping $\phi : G_1 \rightarrow G_2$ is defined by $\phi(A) = det\ A$

i) Show that ϕ is a homomorphism from G_1 to G_2 **(3 Marks)**

ii) Find K_ϕ, the kernel of ϕ **(4 Marks)**

iii) Determine whether ϕ is an isomorphism or not **(3 Marks)**

b) Let G=$\mathbf{Z}_{13}$-{0} under multiplication *mod* 13

i) Evaluate H, the cyclic subgroup generated by 3 **(5 Marks)**

ii) Compute the factor group G/H **(5 Marks)**

QUESTION FIVE (20 MARKS)

a) Given the permutations p=(123)(45) in S_6

i) Determine H the subgroup generated by p **(7 Marks)**

ii) Investigate whether p is odd or even **(3 Marks)**

iii) Calculate p^{-1} **(3 Marks)**

b) Solve for x in the equation

i) $3x^2-2x+2=0 \mod 7$ **(4 Marks)**

ii) $x^2 - x = 6 \mod 7$ **(3 Marks)**