LAIKIPIA UNIVERSITY

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UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN (ICT)

COMP 121: DISCRETE STRUCTURES II

STREAM: R ***TIME: 2 HRS***

DAY: WEDNESDAY [2.30 – 4.30 P.M] ***DATE: 17/04/2024***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

PLEASE DO NOT OPEN UNTIL THE INVIGILATOR SAYS SO.

INSTRUCTIONS: Answer Question ONE and any TWO other questions

QUESTION ONE(COMPULSORY) (30MARKS)

a. Define universal and existential quantification in predicate logic. **[2 Marks]**

b. Explain the difference between modus ponens and modus tollens with examples. **[3 Marks]**

c. Solve there currencerelation $an = 2an-1 + 3$
with the initial condition [3marks] $a0 = 1$ d .

d. Define the terms graph,tree,and spanning tree. **[3 Marks]**

e. Given agraph,describe astrategy to find a spanning tree. **[3 Marks]**

If A= 3 1 4 2 { } B= 7 5 8 6 { }
Compute AB and BA **[3 Marks]**

f. Define the terms: order analysis and standard complexity classes. **[2 Marks]**

g. Given analgorithm ,how would you determine its computational complexity **[2 Marks]**

h. Explain the concept of countability and uncountability. **[2marks]**

i. What are theP and NP classes in the context of computability? **[3 Marks]**

j. Define finite probability spaces, conditional probability, independence, and Bayes' rule. **[4 Marks]**

QUESTION TWO (20 MARKS) *Remember to show your workings.*

Consider a social network where each person is represented as anode,and each friendship is defined as an edge connecting two nodes. The social network can be represented as an undirected graph.

a. Represent this social network as a graph. Assume there are 7 people in the network, and the friendships are as follows: (1,2), (2,3), (3,4), (4,5), (5,6), (6,7), (7,1), (1,3), (2,4),(3,5),(4,6),(5,7),(6,1),(7,2). [2marks]

b. Write a pseudo-code to perform a breadth-first search(BFS)algorithm on this graph startingfromnode1. **[3 Marks]**

c. Find a spanning tree of this graph. **[3 Marks]**

d. Findtheshortestpathbetweennode1andnode5usingKruskal'salgorithm. **[3 Marks]**

e. Determine if the graph contains acycle. If yes,identify the cycle. [3marks]

f. Perform graph coloring of the social network so that no two adjacent nodes have the same color.What is the minimum number of colors you need? [**3 Marks]**

g. Assume the friendships in the net work have acapacity for how muchi nformation can flow through them. The capacity of each edge is 3 units. Find the maximum flow of informationfromnode1(source)tonode7(sink). [**3 Marks]**

QUESTION THREE (20 MARKS)

Consider accompany planning to launch anew software product.The software has no features,and each feature can be implemented by one of the company's team members. Each team has a different skill level and speed of implementation, so the time it takes to implement a feature varies. The company wants to assign the features to the teams to minimize the total implementation time.
You are given a2D array time[i][j]where time[i][j]is the time it takes for team i to implement feature **j.**

a) Formulate this problem as a decision problem and prove it belongs to the class NP. **[4 Marks]**
b) Show that this problem is NP-hard by reducing a known NP-hard problem to it. **[3 Marks]**
c) Propose an algorithm to solve this problem. Discuss its time complexity. **[3 Marks]**
d) Explain why the proposed algorithm might need more efficiency for significant inputs. **[3 Marks]**
e) Propose an approximation algorithm for this problem. Discuss its time complexity. **[4 Marks]**
f) Discuss the approximation ratio of the proposed approximation algorithm **[3 Marks]**

QUESTION FOUR (20 MARKS)

Consider a deterministic Turing machine M that accepts a language L.

a) Define aTuring machine and explain how it can be used to recognize a language. [**2 Marks]**

b) Explain what aTuring machine M means when recognizing a language L. **[3 Marks]**

c) State the halting problem. Prove that it is undecidable. **[4 Marks]**

d) Show that the problem of determining whether a Turing machine M accepts a string w is undecidable by reducing the halting problem to it. **[3 Marks]**

e) Define decidability. Give an example of a decidable problem. [**2 Marks]**

f) State Rice's theorem. Explain why it is essential in the theory of computation. **[4 Marks]**

g) Define NP-completeness Give an example of an NP-complete problem. **[2 Marks]**

QUESTION FIVE (20 MARKS)

consider a city with eight major intersections labeled A, B,C,D,E,F,G,and H.These intersections are connected by roads,forming anetwork we can represent as agraph. In this graph,intersection A is connected to intersections B, C, and D. Intersection B is connected to intersections A, E, and F. Intersection C is connected to intersections A and G. Intersection D is connected to intersections A and H. Intersection E and F are connected and to intersection B. Intersections G and H are only connected tointersectionsCandDrespectively.ThedistancefromintersectionAtoBis5km,AtoC is 3 km, and A to D is 7 km. The distance from intersection B to E is 2 km,andBtoFis4km.The distance from intersection C to G is 6 km, and from D to H is 8 km. Lastly, the distance between intersectionsEandFis1km.

a) Explain the Dijkstra's Algorithm.What is its time complex it y and space complexity? Provide astep-by-step example of how it work so nt hecity's road network. **[5 Marks]**

b) Ifthecityplannedtoconstructanewroadnetworkthatconnectsallintersectionswith the least to tal distance,which algorithmswouldyousuggesttheyuseandwhy? **[3 Marks]**

c) What is the time complexity of the algorithm used in Question B?How doe sit affect the performance of the GPS navigation system as the number of intersections (vertices)increases? **[2 Marks]**

d) If the weights on the edges now represent the average time taken to travel between intersections due to traffic, how would this change your approach in Questions A and C? **[2 Marks]**