LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

1ST SEMESTER 2022/2023 ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE DEGREES OF BACHELOR OF SCIENCE (STATISTICS)/BACHELOR OF SCIENCE (ECONOMICS & STATISTICS)

STAT 312: THEORY OF ESTIMATION

STREAM: R ***TIME: 2 HRS***

DAY: FRIDAY [8.30AM – 10.30AM] ***DATE: 09/12/2022***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS: - Answer question ONE and any other TWO questions.

QUESTION ONE [30 Marks]

a) Define clearly each of the following terms.
   i) Parameter [2 Mark]
   ii) Statistic [2 Mark]

b) Distinguish between an estimator and estimate [3 Marks]

c) What is a point estimator? [2 Marks]

d) State any four requirements of a good estimator [4 Marks]

e) Let $X_1, X_2, ..., X_n$ be independent and identically distributed random variables from a population with unknown mean, θ. Show that the sample mean is an unbiased estimate of θ [3 Marks]

f) If X is a single observation from a distribution with probability density function,

$$f(x) = \begin{cases} \theta x^{\theta-1} & 0 < x < 1 \quad \theta > 1 \\ 0 & elsewhere \end{cases}$$

Use method of moments to estimate θ [5 Marks]

g) Assume that $X_1, X_2, ..., X_n$ are independent and identically distributed $N(\mu, \sigma^2)$ random variables. Find the likelihood function. [4 Marks]

h) Suppose that $X_1, X_2, ..., X_n$ is a random sample from a population with unknown mean (θ) and variance (σ^2). Show that sample mean X is always weakly consistent as an estimator of population mean μ [4 Marks]

i) Let $X_1, X_2, ..., X_n$ be independent and identically distributed random variables from binomial (n, θ). Show that this distribution belongs to a complete family of distributions. [5 Marks]

QUESTION TWO [20 Marks]

a) i) Define likelihood function and state the invariant property of Maximum Likelihood Estimator [4 Marks]

ii) Given that $X_1, X_2, ..., X_n$ are independent and identically distributed Normal(μ, σ^2) random variables. Find the likelihood function [4 Marks]

b) Suppose $X_1, X_2, ..., X_n$ are independent and identically distributed random variables. Find the Maximum Likelihood Estimator given;

i) Bernoulli (θ) [6 Marks]

ii) Geometric distribution with parameter $p,\ 0 \le p \le 1$ [6 Marks]

QUESTION THREE [20 Marks]

a) i) Describe the concept of sufficiency as used in estimation theory [3 Marks]

ii) Suppose that $Y_1, Y_2, ..., Y_n$ are independent and identically distributed Poisson (λ) random variables. Show that $Q = Y_1 + Y_2$ is sufficient for λ [5 Marks]

b) Suppose that $X_1, X_2, ..., X_n$ be independent and identically distributed random samples from normal $N(\mu, \sigma^2)$ where both parameters are unknown. Assume that sample mean $\bar{x}$ is used to estimate μ. Show that $\bar{x}$ is a strong consistent estimator of μ [5 Marks]

c) Let $X_1, X_2, ..., X_n$ be independent and identically distributed $N(\mu, \sigma^2)$ random variables where both μ and σ^2 are unknown. Examine $\overline{X}$ and s^2 for unbiasness as an estimator of μ and σ^2 respectively. [7 Marks]

QUESTION FOUR [20 Marks]

a) Given the model $Y_i = X_i\alpha + \varepsilon, \quad i = 1, 2, ..., n$

i) State its assumptions [2 Marks]

ii) Use method of least squares to estimate α [5 Marks

iii) Assume the model had two parameters, estimate these parameters using method in a)ii) [6 Marks]

b) Suppose $X_1, X_2, ..., X_n$ are independent and identically distributed random variables from Bernoulli (θ) $(1, \theta)$. Show that $f(X : \theta)$ belongs to an exponential family and hence identify a complete and sufficient statistic for θ [7 Marks]

QUESTION FIVE [20 Marks]

a) Explain the concept of prior and posterior distribution as us in Bayesian estimation. [6 Marks]

b) Assume that in a nail manufacturing company the proportion θ of defective nails in a batch is unknown and probability of this θ is uniform distribution in the interval (0, 1). Suppose random samples are taken from the batch such that $X_i = \begin{cases} 1 \text{ if nail is defective} \\ 0 \text{ if nail is non-defective} \end{cases}$ for $X_1, X_2, ..., X_n$ Find the posterior distribution of θ.

[6 Marks]

c) Suppose that p is unknown and that prior distribution of p is $h(p) = \begin{cases} 1 & 0 \le p \le 1 \\ 0 & \text{elsewhere} \end{cases}$ and also that a random variable X has the distribution $f(x|p) = \begin{cases} p(1-p)^{x-1} & x = 1, 2, ... \\ 0 & \text{elsewhere} \end{cases}$. Consider a single observation from the random variable X and find Baye's estimator of p with respect to quadratic loss function. [8 Marks]