



UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

**SECOND YEAR EXAMINATION FOR THE DEGREES OF
BACHELOR OF EDUCATION (ARTS) BACHELOR OF
EDUCATION (SCIENCE) AND BACHELOR OF SCIENCE
(GENERAL)**

MATH 212: VECTOR ANALYSIS

STREAM: R

TIME: 2 HRS

DAY: WEDNESDAY [11.30A.M – 1.30P.M] DATE: 17/04/2024

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTION: ANSWER QUESTION ONE ANY OTHER TWO

QUESTION ONE (30 MARKS)

- a) Classify the following quantities as either vector or scalar; energy, moment, electric fields and volume. **(2 Marks)**
- b) **Given the vectors $\vec{A}=8\hat{i}+2\hat{j}+4\hat{k}$ and $\vec{B}=-3\hat{i}+4\hat{j}-5\hat{k}$: find**
- i) The magnitude of \vec{A} and \vec{B} **(2 Marks)**
 - ii) Directional cosines for \vec{A} and \vec{B} **(2 Marks)**
 - iii) Unit vector in the direction of \vec{B} **(2 Marks)**
 - iv) $\vec{A}+\vec{B}$ **(2 Marks)**
- c) If $\vec{A}=\hat{i}+3\hat{j}-2\hat{k}$ and $\vec{B}=4\hat{i}-2\hat{j}+4\hat{k}$: Find ;
- i) $\vec{A} \cdot \vec{B}$ **(2 Marks)**
 - ii) $|3\vec{A} + 2\vec{B}|$ **(3 Marks)**
 - iii) The angle between \vec{A} and \vec{B} **(3 Marks)**
- d) Find the area of a parallelogram whose diagonals are given by :
- $$\vec{A} = 3\hat{i} + \hat{j} - 2\hat{k}$$
- $$\vec{B} = \hat{i} - 3\hat{j} + 4\hat{k}$$
- (3 Marks)**
- e) A particle moves along a curve whose parametric equations are;
 $x = e^{-t}, y = 2 \cos 3t, z = 2 \sin 3t$, where t is the time determine
- i) Its velocity and acceleration at any time **(2 Marks)**
 - ii) The magnitude of the velocity and acceleration at $t = 0$ **(3 Marks)**
- f) Define the following terms
- i) Solenoidal vector
 - ii) Irrotational vector **(2 Marks)**
- g) Prove that $\vec{A} = 3y^4z^2\hat{i} - 4x^3z^3\hat{j} + 3x^2y^2\hat{k}$ is a solenoidal vector **(2 Marks)**

QUESTION TWO (20 MARKS)

- a) State the Green theorem **(2 Marks)**
- b) Verify the greens theorem in the integral
- $$\oint (xy + y^2) dx + x^2 dy$$

Where C is the closed curve of the region bounded by $y = x$ and $y = x^2$ (6 Marks)

c) Evaluate ;

i) $\iiint_v (\nabla \cdot \vec{F}) dv$ where, $\vec{F} = (2x^2 - 3z)\hat{i} - 2xy\hat{j} - 4x\hat{k}$ and v is the closed volume bounded by the plane $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$ (4 Marks)

ii) $\oiint (\vec{A} \cdot \hat{n}) ds$ where $\vec{A} = 2xz\hat{i} + 2y\hat{j} + yz\hat{k}$ and s is the surface bounded by $x = 0, x = 2, y = 0, y = 1, z = 0, z = 1$ (4 Marks)

iii) $\int_c Q \vec{dr}$ where $Q = 2xyz^2$ and c is the curve given by $x = t^2, y = 2t, z = t^3$ from (0,0,0) to (4,4,8) (4 Marks)

QUESTION THREE (20 MARKS)

a) State Gauss' Divergence Theorem (2 Marks)

b) Given $\vec{A} = x\hat{i} + 2y\hat{j} + z\hat{k}$ and s is the surface bounded by the planes $x = 0, y = 0, z = 0, x + 2y + 2z = 4$. Use Gauss' Divergence theorem to verify (6mks)

c) Find the unit normal to the surface $x^2 + 2y + 2xz = Q$ at the point (2,-3,3) (3 Marks)

d) Given $\vec{A} = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$ find curl \vec{A} at (1,-1,1) (4 Marks)

e) Given $\vec{A} = \cos xy\hat{i} + (3xy - 2x)\hat{j} - (3x + 2y)\hat{k}$; Evaluate

i) $\frac{\partial \vec{A}}{\partial x}$ (2 Marks)

ii) $\frac{\partial^2 \vec{A}}{\partial x \partial y}$ (3 Marks)

QUESTION FOUR (20 MARKS)

a) i) Illustrate the cylindrical coordinate system (r, θ, z) using a diagram (3 Marks)

b) ii) Express r, θ and z purely in terms of x, y and z (3 Marks)

iii) Use cylindrical co-ordinate system to evaluate $\iint_s z^2(x^2 + y^2) dx dy$ where s is the surface bounded by the circle $x^2 + y^2 = 4$ (4 Marks)

c) Given the vectors $\vec{A} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{B} = \hat{i} + 4\hat{j} - 2\hat{k}$. Find;

i) $(\vec{A} \times \vec{B})$ (2 Marks)

ii) $(\vec{B} \times \vec{A})$ (2 Marks)

iii) $(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B})$ (6 Marks)



QUESTION FIVE (20 MARKS)

- a) Given the curve $x = t^2 + 1, y = 4t - 3, z = 2t^2 - 6t$. Find ;
- The unit tangent vector to any point on the curve (4 Marks)
 - Determine the unit tangent at $t = 2$ (2 Marks)
- b) Determine the value of ∇Q at the points $(1, \frac{\pi}{2}, 1)$ given that $Q = Z^2 \cos(xy - \frac{\pi}{4})$ (4 Marks)
- c) Given the vectors $\vec{A} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{B} = \sin t\hat{i} - \cos t\hat{j}$. Find
- $\frac{d}{dt}(\vec{A} \cdot \vec{B})$ (3 Marks)
 - $\frac{d}{dt}(\vec{A} \times \vec{B})$ (3 Marks)
 - $\frac{d}{dt}(\vec{B} \cdot \vec{B})$ (2 Marks)
 - $\frac{d}{dt}(\vec{A} \times \vec{A})$ (2 Marks)

