LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST SEMESTER 2025/2026 ACADEMIC YEAR

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (ECON-STATS) AND BACHELOR OF SCIENCE (STATISTICS)

MATH 221: LINEAR ALGEBRA I

STREAM: R ***TIME: 2 HRS***

DAY: FRIDAY [8.30 – 10.30 A.M] ***DATE: 30/01/2026***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS:

Answer question ONE and ANY other TWO questions

QUESTION ONE (30 MARKS)

a) Given below is a system of linear equations;

$$3x_1+kx_2=4$$
$$kx_1+3x_2=4$$

i) Solve the system when $k=2$ **(3 Marks)**

ii) Determine the value of k for which the system is inconsistent **(4 Marks)**

b) i) Calculate the value of h for which the vectors $v=(-h, 1, 2)$ and $u=(h, -1, h)$ are orthogonal **(3 Marks)**

ii) Evaluate $|\boldsymbol{A}|$ if $|\boldsymbol{A}^{-1}\boldsymbol{B}^2|=6$ and $|\boldsymbol{B}|=2\sqrt{3}$ **(4 Marks)**

c) Given vectors; $\boldsymbol{u}=(1, 5, 0, 0)$, $\boldsymbol{v}=(c, 1, 0, 0)$ and $w=(-1, 1, 0, 0)$ $\in\Re^4$

Evaluate c for which

i) $\|\boldsymbol{u}-\boldsymbol{v}\|=5$ **(4 Marks)**

ii) The vectors u, v and w are linearly dependent **(4 Marks)**

d) Let T be a linear map on a vector space U such that $T(\boldsymbol{u}+\boldsymbol{v})=\alpha$ and $T(3\boldsymbol{v})=2\alpha$, for $\boldsymbol{u}, \boldsymbol{v}$ in U

i) Compute $T(2\boldsymbol{u})$ in terms of α **(4 Marks)**

ii) Evaluate $T(\boldsymbol{x}-\boldsymbol{u})$ if $\boldsymbol{x}$ is in the kernel of T **(4 Marks)**

QUESTION TWO (20 MARKS)

a) The matrix below represents a linear transformation $T: \Re^2 \rightarrow \Re^2$

$$A=\begin{pmatrix} 1 & 3 \\ 2 & 0 \end{pmatrix}$$

i) Evaluate the eigenvalues of A **(3 Marks)**

ii) Compute $\mathbf{X}$ if $\mathbf{A}^2\mathbf{X}=\mathbf{B}$, where $\mathbf{B}=(4, -4)$ **(4 Marks)**

iii) Calculate the determinant of $\mathbf{Q}$ if $|(\mathbf{A}^{-1}\mathbf{Q}^2)|=-2$ **(3 Marks)**

b) The set S={$\boldsymbol{u}$=(2, 2, -3), $\boldsymbol{v}$=(0, -4, 1), $\boldsymbol{w}$=(3, 1, -4)}spans U, a subspace of $\mathfrak{R}^3$

i) Obtain the dimension of U **(5 Marks)**

ii) Investigate whether $\boldsymbol{x}$=(1, -1. -1) is in U or not **(5 Marks)**

QUESTION THREE (20 MARKS)

a) The map T: $\mathfrak{R}^3 \rightarrow \mathfrak{R}^2$ is defined by T(x_1, x_2, x_3)= (2x_1, -x_3)

i) Determine whether T is a linear transformation or not **(5 Marks)**

ii) Compute $\boldsymbol{u}$ in $\mathfrak{R}^3$ such that T($\boldsymbol{u}$)=(2, -3) and $||\boldsymbol{u}||=2\sqrt{3}$ **(5 Marks)**

b) Given below is a collection of vectors in $\mathfrak{R}^4$;

$\boldsymbol{u}$=(2, -1, 1, 0), $\boldsymbol{v}$= (1, 1, c, 1), $\boldsymbol{w}$=(0, 3, 3, 2), $\boldsymbol{x}$=(-1, 2, 1, 1),

i) Evaluate c so that the maximum number of linearly independent vectors is 2 **(5 Marks)**

ii) Express $\boldsymbol{w}$ as a linear combination of $\boldsymbol{x}$ and $\boldsymbol{u}$ **(5 Marks)**

QUESTION FOUR (20 MARKS)

a) Investigate whether the set S is a subspace of V in the following cases

i) S={(x_1, x_2): x_2=2x_1, $x_2 \in \mathfrak{R}$}, V=$\mathfrak{R}^2$

ii) S={(x_1, x_2): x_2= x_1+1, $x_2 \in \mathfrak{R}$}, V=$\mathfrak{R}^2$ **(10 Marks)**

b) The set {$\boldsymbol{u}$=(1, 2, 1), $\boldsymbol{v}$=(1, k+1, 1), $\boldsymbol{w}$=(-2, -4, d)}spans V, a subspace of $\mathfrak{R}^3$. Determine all the values of k and d for which

i) Dimension V is 1

ii) Dimension V is 2 **(10 Marks)**

QUESTION FIVE (20 MARKS)

a) Let T: $\mathfrak{R}^3 \to \mathfrak{R}^2$ be the linear transformation defined by

$$T(x_1, x_2, x_3) = (-x_1, 2x_2 - x_3)$$

i) Show that $\boldsymbol{y}=(0, -1, -2)$ is in the kernel of T. **(3 Marks)**

ii) Evaluate the matrix of T with respect to the standard basis vectors in $\mathfrak{R}^3$ and $\mathfrak{R}^2$ respectively. **(7 Marks)**

iii) State the dimension of the kernel of T. **(2 Marks)**

b) Jane has a number of 20 shilling coins, 50 shilling notes and 100 shilling notes. A combination of any two types of denominations totals to Sh. 600. If the variables x, y and z represent the number of each type of denomination, respectively,

i) Write the system of equations from the given information.

ii) Evaluate x, y and z **(8 Marks)**