LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

FOURTH YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (STATISTICS)

STAT 421: STATISTICAL COMPUTING

STREAM: R ***TIME: 2 HRS***

DAY:THURSDAY[2.30PM-4.30P.M] ***DATE:18/04/2024***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

PLEASE DO NOT OPEN UNTIL THE INVIGILATOR SAYS SO

INSTRUCTIONS

Answer question ONE and any other TWO questions

QUESTION ONE [30 MARKS]

a) Describe types of stochastic simulation models that exist **[4 Marks]**

b) Explain the concept Monte Carlo Simulation and describe how it is performed **[4 Marks]**

c) Suppose in a survey there are fifteen choice questions in a questionnaire and each question has eight possible answers where only one of them is correct. Write an algorithm and hence R commands that computes probability of having four or less correct answers if questions are answered randomly. **[5 Marks]**

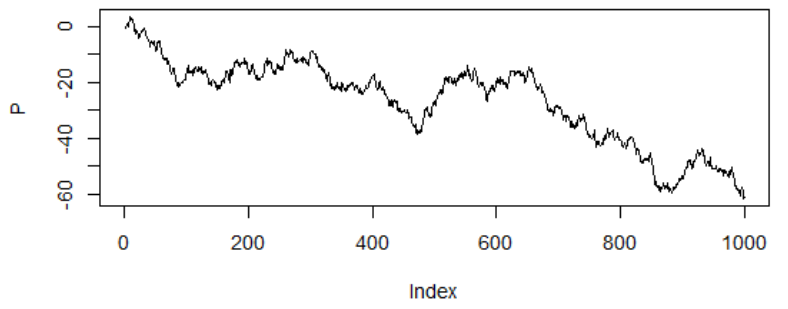
d) Explain the difference between model validation and verification as used in model building. **[4 Marks]**

e) Give an expression for time series $\{x_t : t = 1\}$ and p^{th}-order polynomial linear model **[4 Marks]**

f) Perform transformation of the model $x_t = e^{\alpha_0 + \alpha_1 t + z_t}$ into a linear one **[3 Marks]**

g) Suppose the following commands on the left that were used to simulate random walk data for P gave the output that appears on the right. Explain **[6 Marks]**

```
P <- Q <- rnorm(1000)
for (t in 2:1000) P[t] <- P[t - 1] + Q[t]
plot(P, type = "l")
```



QUESTION TWO [20 MARKS]

a) Consider the following data for X and Y respectively: (1 , 4), (2 , -1), (1.5 , 3), (-2 , 5), (3 , 2), and write program in R that is used to fit a simple linear regression model and determine the SSE using the for loop **[6 Marks]**

b) Given the model $y = \beta_0 + \beta_1 x$, write down an expression for the error of estimation and describe what it means when it is equivalent to zero and when it gets larger. **[4 Marks]**

$$SSE = \sum_{i=1}^{n} \left[y_i - \left(\beta_0 + \beta_1 x \right) \right]$$

c) Given the following density function $f(x) = \begin{cases} \frac{1}{x^2} & \text{if } x \geq 1 \\ 0 & \text{elsewhere} \end{cases}$

Write a program which uses the inverse transform method to generate random numbers. Test your program and write down code to plot a histogram of 10000 random numbers together with the density function. **[10 Marks]**

QUESTION THREE [20 MARKS]

a) Write down and explain briefly R program that would generate one hundred uniform pseudo random numbers given

i) Interval [0,1] and [-25,-10] **[6 Marks]**

ii) Multiplicative congruential generator as **[7 Marks]**

$$x_n = 212x_{n-1} \pmod{240042},\quad u_n = {}^{x_n}\!/_{12042},\quad x_0 = 109575$$

b) Generate ten random numbers using linear congruential generator(LCG) method given that $m = 16, a = 5, c = 3$ and $x_0 = 7$ **[7 Marks]**

QUESTION FOUR [20 MARKS]

a) Simulate a random walk model of length 100, giving appropriate R commands and place simulated data in a vector x given that $x_k = x_{k-0.05} + x_{k+0.95} + m_k$ **[4 Marks]**

b) Write down and explain briefly a program in R that computes 95% confidence interval given mean profit of 15500 **[6 Marks]**

c) Let X be a continuous random variable having the probability density function

$$f(x) = 20x(1-x)^3,\ 0 < x < 1$$

Use rejection method develop an efficient algorithm for simulating the random variable X. **[10 Marks]**

QUESTION FIVE [20 MARKS]

Using appropriate R commands describe how simulation and histogram is plotted. Explain also how a non-linear series model with AR(1) is fitted. **[10 Marks]**

a) Write down with brief explanations R codes that generate the three outputs below following simulation of an AR(1) process for auto regressive model given as $x_t = 0.95x_{t-1} + w_t$

[10 Marks]