LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST SEMESTER 2025/2026 ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (STATISTICS)

MATH 318: CALCULUS II

STREAM: R ***TIME: 2 HRS***

DAY: THURSDAY [14.30 -16.30 P.M] ***DATE: 05/02/2026***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS TO CANDIDATES

- Answer Question **ONE** (COMPULSORY) and ANY **TWO** Questions
- Do not write on the question paper.
- LAIKIPIA UNIVERSITY observes ZERO tolerance to examination cheating.

QUESTION ONE (30 MARKS)

(a). Evaluate the Partial derivatives $f_x(x,y)$ and $f_y(x,y)$ at the point $P_0(x_0,y_0)=(0,0)$ for

$f(x,y)=xe^{-2y}+ye^{-x}+xy^2$ **(4 Marks)**

(b). Evaluate the double integral $\int_0^{\ln 2}\int_{-1}^{0} 2xe^{y}\,dxdy$ **(4 Marks)**

(c). express the decimal 0.252525…... as a fraction, use Geometric series **(4 Marks)**

(d). Use the indicated test for convergence method to determine whether the given series converges or diverges.

(i). $\sum_{k=1}^{\infty}\frac{1}{k^2\sqrt{k}}$ (P-Series test) **(2 Marks)**

(ii). $\sum_{k=1}^{\infty}\frac{(-3)^k}{k^2 2^{2k}}$ (Ratio test) **(4 Marks)**

(iii). $\sum_{k=1}^{\infty}\frac{k}{3+k^2}$ (Integral test) **(4 Marks)**

(e). Find the Taylor series for the function $f(x)=e^{3x}$ at $x=0$ **(4 Marks)**

(f). Suppose the demand function for flour in a certain community is given by

$D_1(p_1,p_2)=500+\frac{10}{p_1+2}-5p_2$, while the corresponding demand for bread is given by

$D_2(p_1,p_2)=400-2p_1+\frac{7}{p_2+3}$, where p_1 and p_2 are prices in shillings for flour and loaf of bread. Determine whether flour and bread are substitutes or complementary commodities or neither. **(4 Marks)**

QUESTION TWO (20 MARKS)

(a) . Determine the radius and the interval of absolute convergence for the power series.

$$\sum_{k=1}^{\infty} \frac{x^k}{\sqrt{k}}$$ **(5 Marks)**

(b). Evaluate the improper integral $\int_{0}^{+\infty} xe^{-2x}dx$ **(7 Marks)**

(c). Use a Taylor polynomial of degree $n = 4$ together with term by term integration to estimate the definite integral $\int_{-0.2}^{0.1} e^{-x^2}dx$ **(8 Marks)**

QUESTION THREE (20 MARKS)

(a) . Find the Limit $\lim\limits_{(x,y)\to(1,1)} \frac{x^2 - 2xy + y^2}{x - y}$ **(3 Marks)**

(b). At what point (x, y) in the plane, are the functions continuous. $f(x, y) = \ln(x^2 + y^2)$

(3 Marks)

(c). Find f_x and f_y, if $f(x, y) = \frac{2y}{y + \cos x}$ **(4 Marks)**

(d). Maximize $x + 2y + 3z$ over the set $x^2 + y^2 + z^2 = 3$ and Maximize $x + 2y + 3z$ over the set $x = 1$. Use the method of Lagrange Multipliers for unconstrained optimization. **(10 Marks)**

QUESTION FOUR (20 MARKS)

(a). Find a power series for the function $f(x) = \frac{x^2}{1 - x^2}$ **(4 Marks)**

(b). Find Taylor series for the function $f(x) = e^{3x}$ at the point $a = 0$ **(4 Marks)** (c). Use Root test to determine whether the given series converges or diverges

$$\sum_{n=1}^{\infty} \frac{n^n}{3^{1+2n}}$$ **(4 Marks)**

(d). Find the critical points of the function $f(x) = xy^2 - 6x^2 - 3y^2$ classify each point as, relative minimum, relative maximum or a saddle point **(8 Marks)**

QUESTION FIVE (20 MARKS)

(a). Use double integral to find the area of R, where R is the region bounded by $y = \frac{1}{2}x^2$ and $y = 2x$ **(6 Marks)**

(b). Find the average value of the function $f(x, y) = xy(x - 2y)$ over the given region $R: -2 \le x \le 3, -1 \le y \le 2$ (you can use double integration) **(6 Marks)**

(c). At a certain factory, output Q is related to inputs x and y by the expression, $Q(x) = 2x^2 + 3x^2y + y^3$. If $0 \le x \le 5$ and $0 \le x \le 7$.

What is the average output of the factory? **(8 Marks)**