LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

1ST SEMESTER 2023/2024 ACADEMIC YEAR

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ECONOMICS & STATISTICS AND BACHELOR OF ECONOMICS & SOCIOLOGY

ECON 214: MATHEMATICS FOR ECONOMISTS III

STREAM: *ECON,STAT* ***TIME: 2 HRS***

DAY: TUESDAY [14.30-16.30 P.M] ***DATE: 5/12/2023***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS

Answer Question ONE (Compulsory) and ANY OTHER TWO questions
Question one carries 30 marks and the rest carries 20 marks each

QUESTION ONE (COMPULSORY)

a) The marginal costs of two firms are given by the following functions:

$C'(Q) = 15 + 20Q - 12Q^2 \quad TC = 100 \; when \; Q = 0$

$C'(Q) = 100e^{0.2Q} \quad TC = 3000 \; when \; Q = 0$

i) Find the total cost function C(Q) for each firm; (4 marks)

ii) Find the average cost function for each firm; (4 marks)

iii) What are the fixed costs of each firm. (2 marks)

b) Find the time path of capital K(t) given the following rates of net investment flow functions;

$(I) = 5t^{1/2} \qquad K(0) = 500$ (3 marks)

$(I) = 18t^{3/5} \qquad K(0) = 30$ (3 marks)

For each of the investment functions in (b) above, find the amount of capital formation over the interval (0,5). (6 marks)

c) Find the producer surplus of the following;

$Q = \sqrt{P - 6}$ given $\bar{P} = 20$ (4 marks)

d) Verify that the following differential equation is exact and solve the equations

$8ytdy + (4y^2 + 3t)dt = 0$ (4 marks)

QUESTION TWO (20 MARKS)

a) Derive general solution of the First order differential equations (FODE). (6 marks)

b) Suppose you are given the following demand and supply functions

$Q_d = \alpha - \beta P \qquad (\alpha, \beta > 0)$

$Q_s = \gamma - \delta P \qquad (\gamma, \delta > 0)$

i) Assuming that the rate of change of price over time is directly proportional to the excess demand, find the time path P(t) (General solution). (6 marks)

ii) What is the inter-temporal equilibrium price; (2 marks)

iii) What is the market clearing equilibrium price; (2 marks)

iv) Does the market have a dynamically stable equilibrium price? Explain. (4 marks)

QUESTION THREE (20 MARKS)

a) Find the general and definite solution to the following differential equations;

$\frac{dy}{dt} + 4y = 10 \qquad y(0) = 12$ (2 marks)

$\frac{dy}{dt} + 10y = 12 \qquad y(0) = 10$ (3 marks)

$\frac{dy}{dt} = 5y - 10 \qquad y(0) = 3$ (3 marks)

b) Find the integral of the following

i. $\int \left(8x^2 e^{(x^3+10)} + \frac{4}{x^3}\right) dx \qquad (x \neq 0)$ (3 marks)

ii. $\int (lnx)^3) dx$ (3 marks)

iii. $\int x \left(x^2 + 3\right)^{(1/2)}\right) dx$ (3 marks)

iv. $\int_2^4 xlnx \ \ dx$ (3 marks)

QUESTION FOUR (20 MARKS)

a) Solve the following difference equations

$y_{t+1} = y_t + 4 \qquad y_0 = 8$ (2 marks)

$y_t = 3y_{t-1} + 4 \qquad y_0 = 1$ (2 marks)

$y_{t+1} = 0.5y_t + 12$ $y_0 = 9$ (2 marks)

$y_{t+1} = 3y_t + 4$ $y_0 = 10$ (2 marks)

b) If the population grows according to the function $H = H_0(2)^{bt}$ and consumption by the function $C = C_0e^{at}$. Find the rates of growth of population of consumption and of per capita consumption by using the natural log. (6 marks)

c) The growing value of GNP is given by:
$GNP_t = GNP_0e^{rt}$ $r = 1.5\%$
   i) If $GNP_0 = 500$, find the value of GNP 10 years from now. (2 marks)
   ii) If $GNP_0 = 1000$, after how many years will the GNP double? (2 marks)

QUESTION FIVE

a) Given $I(t) = 9t^{1/2}$ .Find the level of capital formation in;
   i) 8 years. (2 marks)
   ii) for the fifth through the eighth years (interval [4, 8]). (3 marks)

b) Under a monopoly, the quantity sold and market price are determined by the demand function. If the demand function for a profit-maximizing monopolist is
$P = 274 - Q^2$ and
$MC = 4 + 3Q$,
Find the consumers' surplus. (3 marks)

c) Given the data below,
$C_t = 90 + 0.8Y_{t-1}$ $I_t = 50$ $Y_0 = 1200$; Find
   i) The time path of national income Y_t; (2 marks)
   ii) Check your answer, using t_0 and t_1; (3 marks)
   iii) comment on the stability of the time path. (2 marks)

d) For the data given below,
$Q_{dt} = 180 - 0.75P_t$ $Q_{st} = -30 + 0.3P_{t-1}$ $P_0 = 220$; Determine
   i) The market price P_t in any time period; (2 marks)
   ii) The equilibrium price P_e, and (3 marks)
   iii) The stability of the time path. (2 marks)