



UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

**SECOND YEAR EXAMINATION FOR THE DEGREE OF
BACHELOR OF SCIENCE (STATISTICS)**

MATH 211: CALCULUS II

STREAM: R

TIME: 2 HRS

DAY: MONDAY [8.30A.M – 10.30A.M] DATE: 15/04/2024

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS

Answer all questions in section **A** and any **two** in section **B**

SECTION A**QUESTION ONE (30 MARKS)**

- a) Given that $\int_{12}^{-10} g(x)dx = 6$, $\int_{100}^{-10} g(x)dx = -4$ and $\int_{100}^{-5} g(x)dx = 8$, determine the value of

$$\int_{-5}^{12} g(x)dx. \quad (3 \text{ Marks})$$

- b) Evaluate the integral for the following

(i) $\int (5x^4 + 4x^3 - 2x + 2019)dx$ (2 Marks)

(ii) $\int_0^4 \frac{1}{2}y^{\frac{3}{2}} - \frac{1}{2}y^{-\frac{1}{2}}dx$ (3 Marks)

(iii) $\int_1^5 \frac{2x}{\sqrt{x-1}}dx$ (3 Marks)

(iv) $\int_0^2 \sin^2 \cos x dx$ (3 Marks)

(v) $\int_0^3 \frac{1}{\sqrt{9-t^2}}dt$ (3 Marks)

- c) Find the Maclaurin series expansion of $f(x) = \sin x$. (4 Marks)

- d) Verify the mean-value theorem for $f(x) = x^3 - 3x^2 - 10x + 20$ on the interval $[-1, 5]$. (4 Marks)

- e) Use the trapezoidal rule and the Simpson's rule to approximate the integral

$$\int_0^3 \frac{dx}{1+2x} \text{ using } h = 0.5. \quad (5 \text{ Marks})$$



SECTION B: Answer any TWO questions from this Section.**QUESTION TWO (20 MARKS)**

- a) What is integration? (2 Marks)
- b) Evaluate the integral
- (i) $\int \sin^2 x \cos^2 x dx$ (3 Marks)
- (ii) $\int_{-1}^2 (-x^3 + 5x - 2) dx$ (3 Marks)
- c) Determine all the numbers of c which satisfy the mean value theorem for the function $f(x) = x^3 + 2x^2 - x$ on the interval $[-1, 2]$. (4 Marks)
- d) Evaluate $I = \int_{-10}^3 \int_1^2 \int_1^2 (x + 2y - 2) dx dy dz$ (4 Marks)
- e) Obtain the Taylors series generated by $f(x) = \frac{1}{x}$ at $x = 2$. (4 Marks)

QUESTION THREE (20 MARKS)

- a) Evaluate $\int_0^1 \int_0^1 \int_0^1 x^2 y^2 z^2 dx dy dz$. (4 Marks)
- b) Find $\int_0^{\frac{\pi}{2}} e^x \sin x dx$. (4 Marks)
- c) Find the exact length of the curve $y = \frac{x^4}{8} + \frac{1}{4x^2}$ for $1 \leq x \leq 3$. (4 Marks)
- d) The graph of $y = x^2$ between $x = 1$ and $x = 3$ is rotated completely around the x-axis. Find the volume generated (leave your answer in terms of π). (4 Marks)
- e) Verify Rolle's theorem for $f(x) = x^2 - 2x + 1$ on the interval $[-1, 3]$. (4 Marks)

QUESTION FOUR (20 MARKS)

a) Determine the integral for the following:

(i) $\int 2t^3(1+t^4)dt$ (2 Marks)

(ii) $\int \frac{1}{\sqrt{x^2-2x}} dx$ (3 Marks)

(iii) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$ (3 Marks)

b) Use the trapezoidal rule and the Simpson's rule to approximate $I = \int_0^1 \frac{1}{1+x} dx$ for

$n = 2$. Compare your answer with the exact value. (4 Marks)

c) Decompose $\frac{x+14}{x^2+7x+10}$ into its partial fractions, hence find ;

$\int \frac{x+14}{x^2+7x+10} dx$. (4 Marks)

d) Find the Maclaurin series expansion for $f(x) = e^{-x}$. (4 Marks)

QUESTION FIVE (20 MARKS)

a) If $\frac{dy}{dx} = 2x+1$ and $y = -4$ when $x = 1$;

(i) Find y when $x = -2$. (2 Marks)

(ii) Find x when $y = 0$. (2 Marks)

b) Find $\int \frac{2x^3+3}{x(x-1)^2} dx$. (3 Marks)

c) Find the length of the curve $y = \frac{x^3}{6} + \frac{1}{2x}$ from $x = 1$ to $x = 2$. (3 Marks)

d) If $z = 4x^2y + 3xy^2 + 5x^3y$ determine the second partial derivatives of z hence determine z_{xy} at the point $(2, -1)$. (4 Marks)

e) Find the area of the surface generated by revolving about the x-axis the upper half of the parabola $y^2 = 4x$ between $x = 0$ and $x = 1$. (3 Marks)

f) Calculate $\iint_R \frac{x}{y^2} dx dy$ where $R = [1, 2] \times [4, 6]$. (3 Marks)