LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST SEMESTER 2025/2026 ACADEMIC YEAR

SECOND YEAR EXAMINATION FOR THE DEGREES OF BACHELOR OF SCIENCE (STATISTICS), BACHELOR OF SCIENCE (ECONOMICS AND STATISTICS) AND BACHELOR OF SCIENCE (GENERAL)

STAT 211: MATHEMATICAL STATISTICS I

STREAM: R ***TIME: 2 HRS***

DAY: MONDAY [11.30 – 1.30 P.M] ***DATE: 02/02/2026***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS

- Answer question **ONE** and any other **TWO** questions.

QUESTION ONE (30 MARKS)

a) Suppose X and Y are jointly distributed as follows

$$f(x,y)=\begin{cases}4xy, & 0<x<1,\ 0<y<1\\ 0, & elsewhere\end{cases}$$

Find E(3X + 2Y) **(4 Marks)**

b) If the joint probability distribution function of X and Y is given by

$$f(x,y)=\begin{cases}\frac{1}{36}xy & x=1,2,3\ \ y=1,2,3\\ 0, & elsewhere\end{cases}$$

Find $P[Y \le 2/X=2]$ **(4 Marks)**

c) The joint probability distribution function of X and Y is given by

$$f(x,y)=\begin{cases}\frac{1}{27}(x+y), & x=0,1,2,3\quad y=0,1,2\\ 0, & \text{elsewhere}\end{cases}$$

Find;

i) $P(X \le 2, Y=1)$ **(3 Marks)**

ii) $P(X > 2, Y \le 1)$ **(3 Marks)**

iii) $P(X > Y)$ **(3 Marks)**

iv) $P(X + Y = 4)$ **(3 Marks)**

d) Let X and Y have a bivariate normal distribution with parameters

$\mu_X = 5,\ \mu_Y = 1,\ \sigma_X^2 = 9,\ \sigma_Y^2 = 36,$ and $\rho = \frac{1}{2}$.

Find

i) $P[4 < X < 10.6]$ **(2 Marks)**

ii) $P[3.5 < Y < 9.62 | X = 6]$ **(4 Marks)**

iii) $P[8 < 3X + 5Y < 41]$ **(4 Marks)**

QUESTION TWO (20 MARKS)

a) The joint probability density function of X and Y is given by

$$f(x,y)=\begin{cases} cx^2y, & 0<x<3 \text{ and } 0<y<2 \\ 0 & \text{elsewhere} \end{cases}$$

Find the

i) Value of the constant c **(3 Marks)**

ii) $P(X>1, 1<Y<1.5)$ **(3 Marks)**

iii) Conditional probability distribution $f(y/x=1)$ **(4 Marks)**

b) The joint moment generating function of X and Y is given by
$M(t_1,t_2)=\exp(2t_1+3t_2+2t_1^2-2t_1t_2+8t_2^2)$

Determine;

i) The m.g.f. of X **(1 Mark)**

ii) $E(X)$ **(2 Marks)**

iii) $Cov(X,Y)$ **(7 Marks)**

QUESTION THREE (20 MARKS)

a) Suppose that X and Y are random variables such that $Var(X)=9, Var(Y)=4$ and $\rho_{XY}=-\frac{1}{6}$,

Find $Var(2X+4Y)$. **(4 Marks)**

a) Suppose X and Y have the following joint pmf

Y \ X	1	2	3
1	0.40	0.12	0.08
2	0.15	0.08	0.03
3	0.10	0.03	0.01

Find

i) $E(Y|X=2)$ **(4 Marks)**

ii) $Var(Y|X=1)$ **(4 Marks)**

iii) $E(X|Y=3)$ **(4 Marks)**

iv) $Var(X|Y=1)$ **(4 Marks)**

QUESTION FOUR (20 MARKS)

a) Let X and Y have a bivariate normal distribution with parameters $\mu_x = 7,\ \mu_y = 3,$ $\sigma_x^2 = 9, \sigma_y^2 = 16,$ and $\rho = 0.7$ Find

i) $P[2 < Y < 6]$ **(3 Marks)**

ii) $P[2 < Y < 6 | X = 1]$ **(5 Marks)**

b) The joint probability distribution of two discrete random variables X and Y is given by

$$f(x,y) = \begin{cases} k(2x+3y) & x = 0,1,2 \quad y = 1,2,3 \\ 0, & elsewhere \end{cases}$$

Find;

i) The value of the constant k **(2 Marks)**

ii) Marginal p.d.f. of X and Y **(4 Marks)**

iii) Conditional probability of X given $Y = 2$ **(2 Marks)**

iv) Conditional variance of Y given $X = 1$ **(4 Marks)**

QUESTION FIVE (20 MARKS)

Let X and Y be two random variables with joint p.d.f. given as

$$f(x,y) = \begin{cases} e^{-y} & 0 < x < y < \infty \\ 0, & elsewhere \end{cases}$$

a) Determine the joint mgf of X and Y **(7 Marks)**

b) Find the mgf of X **(2 Marks)**

c) Calculate the correlation between X and Y **(9 Marks)**

d) Are X and Y independent? **(2 Marks)**