LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST SEMESTER 2025/2026 ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (ECONOMICS & STATISTICS)

STAT 322: HYPOTHESIS TESTING

STREAM: R ***TIME: 2 HRS***

DAY: TUESDAY [8.30 – 10.30 A.M] ***DATE: 03/02/2026***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Explain the following terms;

i) Size α test

ii) Test of a statistical hypothesis **(4 Marks)**

b) Let the p.d.f of a random variable X is;

$$f(x) = \frac{1}{\theta} \quad , \quad 0 < x < \theta$$
$$= 0 \quad , \quad elsewhere$$

We wish to test $H_0 : \theta = \frac{3}{2}$ against $H_1 : \theta > \frac{3}{2}$. Suppose we have a random sample of one observation and the critical region is $C = \{x : x > 1.2\}$. Find;

i) The significance level of the test.

ii) The power function $\pi(\theta)$ and hence or otherwise $\pi(2)$ and $\pi(2.5)$. **(5 Marks)**

c) Consider a random sample $X_1, X_2, X_3, ..., X_n$ from a normal distribution $N(\theta_0, 36)$. We wish to test the hypothesis $H_0 : \theta = \theta_0$, against the alternative $H_1 : \theta \neq \theta_0$. Show that the likelihood ratio, λ is given as;

$$\lambda = \frac{1}{\left\{1 + \left[\frac{n\left(\overline{X} - \theta_0\right)^2}{\sum\left(X_i - \overline{X}\right)^2}\right]\right\}^{\frac{n}{2}}}$$

(7 Marks)

d) Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from a $N(\theta, 100)$ distribution where the parameter $\theta \in \Re$ is unknown. Find the most powerful test of $H_0 : \theta = 75$ versus $H_1 : \theta = 78$. **(7 Marks)**

e) Let $X_1, X_2, \ldots, X_n$ denote a random sample of size n from a geometric (θ) distribution where $\theta \in (0,1)$ is the unknown parameter. Derive a uniformly most powerful test of size α for testing $H_0 : \theta = \frac{1}{2}$ versus $H_1 : \theta > \frac{1}{2}$. **(7 Marks)**

QUESTION TWO (20 MARKS)

a) Distinguish between simple and composite hypothesis citing examples in each case **(3 Marks)**

b) State and proof the Neyman-Pearson lemma. **(7 Marks)**

c) Briefly describe a uniformly most powerful test. **(4 Marks)**

d) Let y represent a single observation from a population with probability density function given by

$$f\left(y/\theta\right)=\begin{cases}\theta y^{\theta-1}, & 0<y<1\\ 0, & eslewhere\end{cases}$$

Find the most powerful test with significance level α =0.05 to test $H_0:\theta=2$ vs $H_1:\theta=1$.

(6 Marks)

QUESTION THREE (20 MARKS)

a) Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from $N(\mu,\sigma^2)$ distribution where $\mu\in\Re$ is known but $\sigma\in\Re^+$ is assumed unknown. Derive a $(1-\alpha)$ confidence interval for σ by inverting the likelihood ratio test of $H_0:\sigma^2=\sigma_0^2$ versus $H_1:\sigma^2\neq\sigma_0^2$. **(5 Marks)**

b) Let $X1, \ldots, Xn$ denote an independent random sample from a population with a Poisson distribution with mean λ. Derive the most powerful test for testing H_0: $\lambda = 2$ versus H_1: $\lambda = 1/2$.

(7 Marks)

c) Let $X_1, X_2, X_3, ..., X_{20}$ be a random sample of size $n=20$ from a normal population with unknown mean μ and variance $\sigma^2=5$.

We wish to test $H_0:\mu=7$ vs $H_1:\mu>7$.

i. Find the uniformly most powerful test with significance level of 0.05

ii. For the test in part (i), find the power at each of the following alternative values for $\mu:\mu_1=7.5, 8.0, 8.5$ and 9.0

iii. Sketch the power function **(8 Marks)**

QUESTION FOUR (20 MARKS)

a) Briefly describe the generalized likelihood ratio test. **(5 Marks)**

b) Let $X_1, X_2, X_3, ..., X_n$ be a random sample from an exponential distribution with parameters θ where, $\Theta = \{\theta;\ \theta > 0\}$. Derive the likelihood ratio test for $H_0 : \theta \le \theta_0$ vs $H_1 : \theta > \theta_0$. **(8 Marks)**

c) Let $X_1, X_2, \ldots, X_8$ be a random sample of size $n = 8$ from a probability distribution function $f(x;\lambda) = \dfrac{\lambda^x e^{-\lambda}}{x!}, x = 0,1,2,.......$, where $\theta > 0$ is the unknown parameter. Find the most powerful test of size α of $H_0 : \theta = 5$ versus $H_1 : \theta = 15$ in the simplest form. **(7 Marks)**

QUESTION FIVE (20 MARKS)

a) Use the MLR approach to derive the UMP level α test of $H_0 : \theta = \theta_0$ versus $H_1 : \theta > \theta_0$ for a random sample of size n from a Bernoulli (θ) distribution, where $\theta \in (0,1)$ is the unknown parameter.

(7 Marks)

b) Take $n = 10$, $\theta_0 = 0.2$ and $\alpha = 0.033$, and sketch the power function of the UMP test in part (a)

(6 Marks)

c) Let $X_1, X_2, \ldots, X_8$ be a random sample of size $n = 8$ from a Poisson distribution with mean θ. To test $H_0 : \theta = 0.5$ versus $H_1 : \theta > 0.5$, the following test was used: Reject H_0 if and only if $\sum_{i=1}^{8} X_i \ge k$.

i) Find the power function $\pi(\theta)$ as a sum of Poisson probabilities

ii) Determine the value of k such that the test is size 0.01. **(7 Marks)**