LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

1ST SEMESTER 2022/2023 ACADEMIC YEAR

SECOND YEAR EXAMINATION FOR THE DEGREES OF BACHELOR OF SCIENCE (STATISTICS)/BACHELOR OF SCIENCE (ECONOMICS & STATISTICS)/ BACHELOR OF SCIENCE (ICT)/BACHELOR OF SCIENCE (COMPUTER SCIENCE)/BACHELOR OF SCIENCE (GENERAL)

STAT 211: MATHEMATICAL STATISTICS I

STREAM: R ***TIME: 2 HRS***

DAY: WED [8.30AM – 10.30AM] ***DATE: 21/12/2022***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES
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INSTRUCTIONS: - Answer question one and any other two questions

QUESTION ONE (30 MARKS)

a) Let X and Y be two jointly continuous random variables with joint pdf

$$f(x,y)=\begin{cases}10x^2y, & 0<y<x<1\\ 0, & elsewhere\end{cases}$$

i) Find the marginal pdfs (4 marks)

ii) Find $P\left(Y<\frac{1}{4}/X=\frac{1}{2}\right)$ (3 marks)

b) Given the joint density function

$$f(x,y)=\begin{cases}kxy, & 0<x<1, \quad 0<y<2\\ 0, & elsewhere\end{cases}$$

i) Find the value of k. (3 marks)

ii) Determine $\Pr(0\le X\le \frac{1}{2},\ \frac{1}{3}\le Y\le \frac{3}{2})$ (4 marks)

iii) Determine $\Pr(0\le X\le \frac{1}{2}/\ Y=\frac{1}{3})$ (4 marks)

c) Let X and Y have the bivariate normal distribution with

$\mu_x=3,\ \mu_y=1,\ \sigma_x^2=16,\ \sigma_y^2=25$ and $\rho_{xy}=\frac{3}{5}$. Find;

i) $P(3<Y<8)$ (2 marks)

ii) $P(3<Y<8/X=7)$ (4 marks)

d) Let X and Y have the joint p.d.f $f(x,y)=\begin{cases}2, & 0<x<y<1\\ 0, & elsewhere\end{cases}$. Find

$Var\left(X\middle|Y=\frac{1}{2}\right)$ (6 marks)

QUESTION TWO (20 MARKS)

a) Suppose that the two random variables X and Y have the following probability distribution:

		X			
		0	**1**	**2**	**3**
Y	**0**	0	0.02	0.07	0.1
	1	0.02	0.07	0.1	0.1
	2	0.07	0.15	0.1	0.2

(i) Show that P(X, Y) is a valid probability distribution function (2 marks)

(ii) Are X and Y independent. Justify your answer (3 marks)

(iii) Find the marginal distribution of X (2 marks)

(iv) Find P(X<2/Y=1) (3 marks)

b) A soft drink machine has a random amount Y supplied to it at the beginning of a given day and dispenses a random amount X during the day in gallons. It is not resupplied during the day. It has been observed that X and Y have a joint density function given by:

$$f(x,y) = \begin{cases} \frac{1}{2}, & 0 \le x \le y, \quad , \quad 0 \le y \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

i. Calculate the probability that at most a half a gallon is sold given the machine contains one gallon at the start of the day (6 marks)

ii. Are X and Y dependent? (4 marks)

QUESTION THREE (20 MARKS)

a) Let X and Y have a bivariate normal distribution with parameters $\mu_x = 2.9$, $\mu_y = 2.4$, $\sigma_x = 0.4$, $\sigma_y = 0.5$ and $\rho_{xy} = 0.5$. Determine the following probabilities

i) $P(2.1 < Y < 3.3)$ (3 marks)

ii) $P(2.1 < Y < 3.3 / X = 3.2)$ (5 marks)

iii) $P(-3.1 < X < 4.3)$ (3 marks)

iv) $P(-3.1 < X < 4.3 / Y = 2)$ (5 marks)

b) Suppose that X and Y are random variables such that $Var(X) = 9, Var(Y) = 4$ and $\rho_{XY} = -\frac{1}{3}$. Find $Var(3X + 2Y)$. (4 marks)

QUESTION FOUR (20 MARKS)

a) Suppose X and Y are jointly distributed as follows

$$f(x,y) = \begin{cases} \frac{k}{2}xy, & 0 < x < 1, \ 0 < y < 1 \\ 0, & elsewhere \end{cases}$$

Find;

i) The value of k (4 marks)

ii) E(XY) (4 marks)

b) The joint probability mass function of X and Y is given by

$$f(x,y)=\begin{cases}\dfrac{3x+y}{8c}, & x=1,2 \quad y=1,2\\ 0, & elsewhere\end{cases}$$

Find:

i) The value of c. (4 marks)

ii) The conditional variance $Var(X/Y=2)$ (8 marks)

QUESTION FIVE (20 MARKS)

Let X and Y be two random variables with joint p.d.f. given as

$$f(x,y)=\begin{cases}e^{-y} & 0<x<y<\infty\\ 0, & elsewhere\end{cases}$$

a) Determine the joint mgf of X and Y (7 marks)

b) Find the mgf of X (2 marks)

c) Calculate the correlation between X and Y (9 marks)

d) Are X and Y independent? **(2 marks)**