



UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE DEGREE OF
BACHELOR OF EDUCATION (SCIENCE/ARTS) AND BSC
(GENERAL)

MATH 325: NUMERICAL ANALYSIS 1

STREAM: R

TIME: 2 HRS

DAY: FRIDAY [11.30 A.M – 1.30P.M]

DATE: 19/04/2024

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS: Answer Question **ONE** and Any Other **TWO** Questions

QUESTION ONE (30 MARKS)

(a). Find the approximate root of $x^3 - 5x - 1 = 0$ using the method of fixed-point iteration **(5 Marks)**

(b). (i). Write an iteration for finding $\frac{1}{N}$ where $N > 0$. **(3 Marks)**

(ii). Hence evaluate $\frac{1}{26}$ correct to four decimal places **(4 Marks)**

(c). Using Newton's divided difference interpolation, find $y(10)$ for the data
 $y(5) = 12, y(6) = 13, y(9) = 14, y(11) = 16$ **(4 Marks)**

(d). Find $\frac{dy}{dx}$ at $x = 1$ from the following table of values **(4 Marks)**

x	1	2	3	4
y	1	8	27	64

(e). Prove that $\Delta - \nabla = \Delta \nabla$ **(3 Marks)**

(f). (i). Evaluate the integral $\int_0^1 xe^x dx$ using Simpson's $\frac{1}{3}$ rule and compare with the actual value **(6 Marks)**

(ii). What is the number of nodal points required for using Simpsons $\frac{1}{3}$ rule for integrating

$$\int_a^b f(x) dx$$

(1 Mark)

QUESTION TWO (20 MARKS)

(a). Find an approximate root for the equation $xe^x = 2$ using the method of False Position. **(8 Marks)**



(b). For the data below construct the Newton's forward interpolation formula. Hence find $f(0.5)$

(4 Marks)

x	-2	-1	0	1	2	3
y	15	5	1	3	11	25

(c). Derive the Newton's forward interpolation formula using the operator relations

(4 Marks)

(d). Using Lagrange interpolation formula find $y(10)$ from the following table

x	5	6	9	11
y	12	13	14	16

(4 Marks)

QUESTION THREE (20 MARKS)

(a). Find the n th term of the sequence 1, 4, 10, 20, 35, 56, Also find the 7th term. (8 Marks)

(b. (i). Find the approximate value of $I = \int_0^1 \frac{dx}{1+x}$ using the trapezium rule, with 2, 4, 8 equal subintervals

(6 Marks)

(ii). Apply Romberg's method to question 3.b.(i) to improve the approximation, compare with the actual value

(6 Marks)

QUESTION FOUR (20 MARKS)

(a). Using the operator relations, derive the approximations to the derivatives $f'(x_n)$ and $f''(x_n)$ in terms of the backward differences. (8 Marks)

(b). Find $f'(2.5)$ and $f''(2.5)$ using the Newton's backward difference method for the data of the function $f(x) = e^x + 1$ (12 Marks)

x	1.0	1.5	2.0	2.5
$f(x)$	3.7183	5.4817	8.3891	13.1825

QUESTION FIVE (20 MARKS)

(a). Assuming that a root of $x^3 - 9x + 1 = 0$ lies in the interval(2,4), find the root by bisection method.

(6 Marks)

(b). The velocity of a particle which starts from rest is given by the following table.

t(sec)	0	2	4	6	8	10	12	14	16	18	20
v(ft/sec)	0	16	29	40	46	51	32	18	8	3	0

Evaluate using Simpsons 1/3 rule, the total distance travelled in 20 seconds. **(6 Marks)**

(c). The value 7.36 is as correctly rounded approximation to x . Obtain as correctly as possible an approximation \sqrt{x} . **(5 Marks)**

(d). Convert $(21)_{10}$ to binary number system **(3 Marks)**