LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (STATISTICS & ECONOMICS)

STAT 223: MATHEMATICAL STATISTICS II

STREAM: R ***TIME: 2 HRS***

DAY: TUESDAY [8.30 – 10.30 A.M] ***DATE: 16/04/2024***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

PLEASE DO NOT OPEN UNTIL THE INVIGILATOR SAYS SO

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO

QUESTION ONE [30 MARKS]

a) Explain the following laws of large numbers
   i) Weak Law of Large Numbers **(3 Marks)**
   ii) Strong Law of Large Numbers **(3 Marks)**

b) Consider the following joint probability density function

$$f(x_1, x_2, x_3) = \begin{cases} k(x_1 + 2x_2 + 3x_3) & 0 < x_i < 1 ;\ i = 1,2,3 \\ 0 & elsewhere \end{cases}$$

Determine;
   i) The constant k **(3 Marks)**
   ii) The marginal probability density function of X_1 and X_2 **(4 Marks)**

c) Let X and Y have the joint probability density function given by;

$$f(x,y) = \frac{x+2y}{18}, x = 1,2. \quad y = 1,2.,$$

Find the mean of μ_x and μ_y **(6 Marks)**

d) Given the joint probability mass function

$$f(x_1, x_2, x_3) = \begin{cases} \frac{1}{108} x_1 x_2 x_3 & \text{where} \quad x_1 = 1,2,3; x_2 = 1,2,3; \text{and } x_3 = 1,2 \\ 0 & otherwise \end{cases}$$

i) Find the conditional distribution of X_3 given that X_1=1 and X_2=2. **(6 Marks)**
ii) Find the conditional distribution of X_3 and X_2 given that X_1=3. **(5 Marks)**

QUESTION TWO (20 MARKS)

e) Find $f_{UW}(U, W)$, if $U = X^2 + Y^2$ and $W = X^2$ **(10 Marks)**
f) Let joint distribution function of X and Y is given by;

$$f(x,y) = \begin{cases} xe^{-x(y+1)} & , x \geq 0, y \geq 0 \\ 0 & otherwise \end{cases},$$

Find the conditional probability density functions of X and Y **(10 Marks)**

QUESTION THREE (20 MARKS)

a) The joint CDF of two discrete random variables X and Y is given by;

$$F_{XY}(XY)\begin{cases}\frac{1}{8}, & x=1, y=1\\ \frac{5}{8}, & x=1, y=2\\ \frac{1}{4}, & x=2, y=1\\ 1, & x=2, y=2\end{cases}$$

Find;

i) The joint marginal probability mass functions of X and Y **(6 Marks)**

ii) The marginal probability density functions of X **(2 Marks)**

iii) The marginal probability density functions of Y **(2 Marks)**

b) Let X ~ N (μ, σ^2). Derive the characteristic function of X. **(10 Marks)**

QUESTION FOUR (20 MARKS

Assume that the random variable$X_1, X_2 \ldots X_{10}$ are independent and identically distributed with the common PDF $f_x(X)$ and common CDF $F_x(X)$. Find the PDF and CDF of the following;

i) the 3rd largest random variable **(5 Marks)**

ii) the 5th largest random variable **(5 Marks)**

iii) the largest random variable **(5 Marks)**

iv) the smallest largest random variable **(5 Marks)**

QUESTION FIVE [20 MARKS]

a) Suppose X_1, X_2, X_3, and X_4 have the 4-variate joint probability distribution function given by;

$$f(x_1x_2x_3x_4) = \begin{cases}16x_1x_2x_3x_4 & 0 < x_i < 1 \quad \forall i = 1,2,3,4\\ 0 & elsewhere\end{cases}$$

Determine the marginal probability density function of;

i) X_2, and X_4 **(3 Marks)**

ii) X_3 **(3 Marks)**

b) Using the central limit theorem, calculate the approximate value of $P(0.3 \leq Y \leq 1.5)$ **(5 Marks)**

c) The joint PMF of two random variables X and Y is given by;

$$P_{XY}(x,y) = \begin{cases} k(2x+y), x = 1,2, y = 1,2. \\ 0 \quad\quad otherwise \end{cases}$$

Find;

i) The value of k **(3 Marks**

ii) The marginal probability density functions of X and Y **(6 Marks)**