LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (GENERAL)

MATH 318: CALCULUS III

STREAM: R ***TIME: 2 HRS***

DAY: TUESDAY [8.30A.M – 10.30A.M] ***DATE: 16/04/2024***

THIS QUESTION PAPER CONSISTS OF FOUR (4) PAGES

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INSTRUCTIONS

Answer all questions in section A and any TWO in section B

SECTION A

QUESTION ONE (30 MARKS)

a) Determine the partial derivatives of the following;

(i) $z = x^2 + 2xy$ **(2 Marks)**

(ii) $z = xy\tan xy$ **(3 Marks)**

(iii) $z = e^{\sin xy}$ **(3 Marks)**

b) Find the limits of the following sequences if they do converge

(i) $\left\{ \dfrac{3n^2 - 1}{10n + 5n^2} \right\}_{n=2}^{\infty}$ **(3 Marks)**

(ii) $\left\{ \dfrac{(-1)^n}{n} \right\}_{n=1}^{\infty}$ **(2 Marks)**

c) State the D'Alembert's ratio test for convergence and use it to test if $\sum_{n=1}^{\infty} \dfrac{e^{-n}}{n+1}$ converges or diverges. **(5 Marks)**

d) Using geometric series, write the indefinite number $3.3333...$ as a rational number. **(3 Marks)**

e) Find all local maxima and minima of $f(x, y) = x^2 + xy + y^2 - 3x$. **(5 Marks)**

f) Find the volume beneath the surface $f(x, y) = 18 - 3y$ and above the domain bounded by $y = 2x$ and $y = x^2$. **(4 Marks)**

SECTION B: ANSWER TWO QUESTIONS IN THIS SECTION

QUESTION TWO (20 MARKS)

a) Find the total differential dw given that $w=\sqrt{x^2+y^2+z^2}$ **(4 Marks)**

b) Define the limit of a sequence. Hence evaluate the following limits; **(2 Marks)**

(i) $\lim_{n\to\infty}\sqrt[n]{3^{n+1}}$ **(2 Marks)**

(ii) $\lim_{n\to\infty}\left(\frac{1}{n}\right)^{-\frac{4}{n}}$ **(2 Marks)**

c) Evaluate $\int_0^1\int_0^x 2e^{x^2}\,dydx$ **(3 Marks)**

d) Use the method of Lagrange multipliers to find the minimum value of

$f(x,y)=x^2+4y^2-2x+8y$ subject to the constraint $x+2y=7$. **(7 marks)**

QUESTION THREE (20 MARKS)

a) Define the following terms;

(i) Infinite series. **(2 Marks)**

(ii) Convergence of a sequence. **(2 Marks)**

b) Use the method of differential to approximate the value of $(1.02)^3(0.97)^2$ given that $x=1$ and $y=1$. **(4 Marks)**

c) Use comparison test to examine the convergence $\sum_{n=1}^{\infty}\frac{n^2+2}{n^4+5}$. **(4 Marks)**

d) Determine and classify the critical points of $g(x,y)=\frac{1}{3}x^3+y^2+2xy-6x-3y+4$.

(8 Marks)

QUESTION FOUR (20 MARKS)

a) Show that the equation $f(x,y)=x^2+xy+y^2$ satisfies the Euler's theorem

$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=nf(x,y)$. **(3 Marks)**

b) Given an implicit function of the form $f(x,y)=\left(x^2+y^2\right)^3-3\left(x^2+y^2\right)+1$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. **(4 Marks)**

c) Find the equation of the tangent plane and the equation of the normal to the surface $x^2+y^2+z=2z$ at the point $(\cos\beta,\sin\beta,1)$. **(6 Marks)**

d) Evaluate $\iint e^{2x+3y}dxdy$ over the triangle bounded by the lines $x=0, y=0$ and $x+y=1$.

(4 Marks)

e) Find the limit of the sequence $\left(\frac{2-n^2}{3+n}\right)^n$ and confirm its convergence. **(3 Marks)**

QUESTION FIVE (20 MARKS)

a) State the second derivative test for functions of two variables. **(5 Marks)**

b) Evaluate the integral $\iiint_B 8xyz\,dV$, $B=[2,3]\times[1,2]\times[0,1]$. **(3 Marks)**

c) Determine the general term of the series $\frac{3}{4}+\frac{5}{4^2}+\frac{7}{4^3}+\ldots$ **(2 Marks)**

d) A golf ball manufacturer, Pro-T has developed a profit model that depends on the number of x golf balls sold per month (measured in thousands), and the number of hours per month of advertising y, according to the function $z=f(x,y)=48x+96y-x^2-2xy-9y^2$, where z is measured in thousands of dollars. The budgetary constraint function relating the cost of production of thousands of golf balls and advertising units is given by $20x+4y=216$. Find the values of x and y that maximize profit, and find the maximum profit. **(10 Marks)**