LAIKIPIA UNIVERSITY

UNIVERSITY EXAMINATIONS

1ST SEMESTER 2022/2023 ACADEMIC YEAR

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (ECONOMICS & STATISTICS)

STAT 322: TESTING OF HYPOTHESIS

STREAM: R ***TIME: 2 HRS***

DAY: THURS [8.30AM – 10.30AM] ***DATE: 08/12/2022***

THIS QUESTION PAPER CONSISTS OF FIVE (5) PAGES

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<u>INSTRUCTIONS</u>: Answer question **ONE** and any **TWO** questions

QUESTION ONE (30 MARKS)

a) Briefly explain the differences between the following concepts: -

(i) Statistical hypothesis and test of hypothesis. (2 marks)

(ii) Simple and composite hypotheses (2 marks)

(iii) Type 1 and Type 11 errors. (2 marks)

(iv) Most powerful and uniformly most powerful tests. (2 marks)

b) A manufacturer of light bulbs has to decide on basis of sample whether 65% of all products sold will not be faulty, $H_o : \Theta = 0.65$ against $H_1 : \Theta = 0.80$, the test statistic X, the observed number of success in n = 10 trials and will accept the H_o if $x \geq 6$ otherwise will conclude that $\Theta = 0.80$. Compute probability of

i) Type 1 error. (4 marks)

ii) Power of the test. (4 marks)

c) Given the probability distribution:

$$f(x,\Theta) = \begin{cases} \frac{1}{2\theta} & 0 \leq x \leq \theta \\ 0 \text{ } elsewhere & \end{cases}$$

Suppose that it is desired to test the null hypothesis $H_o : \Theta = 7$ against $H_1: \Theta = 10$ by means of a single observed value of x. What would be the size of type 1 and type 11 errors if you choose the interval?

i) $0.05 \leq x$

ii) $1 \leq x \leq 1.75$ (8 marks)

d) An experimenter is concerned that the variability of responses using two different procedures may not be the same. Before conducting his research, he conducts a prestudy with random samples of 10 and 8 responses and gets $s_1^2 = 7.14$ and $s_2^2 = 3.21$, respectively. Using $\alpha = 0.05$ construct point estimate interval for the ratio of the two population variances.

Do the sample variances present sufficient evidence to indicate that the population variances are unequal? (6 marks)

QUESTION TWO (20 MARKS)

a) Suppose that $X_1, X_2, X_3 \ldots\ldots\ldots\ldots, X_n$ is a random sample from a normal population with mean μ and variance σ^2 , both unknown. Derive a test procedure for testing the hypothesis: $H_o: \mu = \mu_0$ against $H_1: \mu \neq \mu_0$.

(12 marks)

b) Mr. Faida an investment analyst gathered the following data on the 91-day Treasury bill rates for the year 2009 and 2010

Month	Treasury bill rates (%)2006	Treasury bill rates (%)2007
January	3.2	5.5
February	3.0	5.2
March	2.8	4.3
April	2.5	3.6
May	2.9	3.3
June	3.4	2.7
July	3.7	2.4
August	4.0	2.0
September	3.8	2.3
October	4.2	2.8
November	4.5	3.1
December	5.1	3.7

Determine if there is a significant difference in the average Treasury bill rates. (Use $\alpha = 0.05$). (8 marks)

QUESTION THREE (20 MARKS)

a) i) State without proof the Neyman-Pearson Lemma. (3 marks)

ii) Let x be a normal random variable with known mean μ and unknown variance σ^2. Consider the hypothesis $H_o: \sigma = \sigma_0$ against $H_1: \sigma \neq \sigma_1$, where σ_0 and σ_1 are known specified values.

I. Find the powerful critical region (8 marks)

II. Power of the test (3 marks)

b) i) If probabilities of Type 1 error and Type 11 error are α and β respectively show that the required size of the sample (n) is given by

$$n = \frac{\sigma^2 (Z_\alpha + Z_\beta)^2}{(\mu_1 - \mu_0)^2}$$ (4 marks)

ii) Find the required size of the sample size when $\sigma=9$, $\mu_0=15$, $\mu_1=20$, $\alpha=0.05$ and $\beta=0.01$.

(2 marks)

QUESTION FOUR (20 MARKS)

a) Briefly describe how one can achieve an ideal test based on type 1 and type 11 error.

(4 marks)

b) Let X be normally distributed, let $x \sim N(\mu, 196)$ with size 25 at $\alpha = 0.05$. We wish to test the hypothesis H_o: $\mu = 97$ against H_1: $\mu = 102$. FIND: -

I. Critical region

II. Power of the test (4 marks)

c) A textile fiber manufacturer is investigating a new drapery yarn, which the company claims has a mean thread elongation of 12 kilograms with a standard deviation of 0.5 kilograms. The company wishes to test the hypotheses H_o: $\mu = 12$ against H_1: $\mu \neq 12$ using a random sample of four specimens.

i) What is the type 1 error probability if the acceptance region is defined as $11.59 < \bar{X} < 12.51$ kilograms? (3 marks)

ii) Find the power of the test for detecting a true mean elongation is 11.25 kilograms using the acceptance region as defined in (i) (3 marks)

d) Professor Jack Ma has recently employed a new gate keeper who is expected to open the gate of the entrance path to his home every time the professor returns from work. Over 12 working days he records the number of times he has to ring the bell until the gate keeper opens the gate. The records are as follows; 7,4,10,1,5,2,8,0,3,7,2,6

Does he have good reason to replace the gate keeper? Note that a smart gate keeper respond to the first ring of the bell. (Use $\alpha = 0.05$). (6 marks)

QUESTION FIVE (20 MARKS)

a) Suppose that $X_1, X_2, X_3 \ldots\ldots, X_n$ is a random sample from a normal population with mean μ and variance σ^2, both unknown. Consider the hypothesis H_o: $\mu = \mu_0$ against H_1: $\mu \neq \mu_0$. Construct the likelihood ratio test (LRT) hence obtain λ.

(12 marks)

b) The table below shows the marks awarded to nine children in a competition.

Child	A	B	C	D	E	F	G	H	I
Judge 1	6.8	7.3	8.1	9.8	7.1	9.2	8.1	8.2	7.3
Judge 2	7.8	9.4	7.9	9.6	8.9	6.9	7.8	7.9	9.4

i) Compute the Spearman's rank correlation coefficient for the data and comment on the value obtained. (4 marks)

ii) Using the hypotheses HO: $\rho = 0$, no correlation exists
Against H_1: $\rho \neq 0$, correlation exists. Compute the test statistic (2 marks)

Determine whether there is any significant correlation in the voting by the readers and critics, test at 0.05 level of significance. (2 marks)